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# 1 Charlas Plenarias / Plenary Talks

## The Mathematics of Music and Art

Arturo Portnoy, University of Puerto Rico at Mayagüez.

Music, mathematics and science have a long, joint history. Each discipline has both benefited from and provided impulse to the others. The exceedingly universal appeal of music provides powerful motivation to study both mathematics and science. Through the lens of mathematics and science, music takes on a more cerebral and analytic dimension, which is both practical and powerful. An undergraduate, no prerequisite course and a new textbook will be presented and personal experiences teaching the course will be shared. The tremendous pedagogical potential in exploiting the synergy between these disciplines will be explored and illustrated.

## AI History, Breakthroughs and its Impact on Scientific Research

Rémi Mégret, University of Puerto Rico at Río Piedras.

This presentation aims to provide a comprehensive overview of the background leading to the recent explosion of artificial intelligence (AI) and its profound impact on scientific research. By exploring key methodological milestones, we will offer a detailed perspective on modern AI and valuable insights into its core components.

This historical perspective will highlight the profound changes in approach from the inception of AI and early models focused on algorithmic and symbolic reasoning, to the rise of machine learning and data-driven approaches that led to the success of neural network approaches at the core of deep learning, generative AI models and now agentic AI and reasoning systems. Each of these steps is associated with significant methodological innovations which we will explore by looking at the inner working of their respective approaches.

We will review some of the significant contributions of AI on various scientific disciplines, to illustrate the large diversity of approaches in which AI fosters new discoveries. This will allow us to examine various aspects that are central to its success, such as the availability of good quality data or feedback, the advances in machine learning models and dedicated cyberinfrastructure. Additionally, we will discuss the major risks associated with the accelerated growth of AI technology and its widespread use in academia and society.

# Inverse Probability (Bayesian Statistics) in Puerto Rico: The First 25 Years

*Luis R. Pericchi Guerra*, University of Puerto Rico at Río Piedras.

At the Biostatistics and Bioinformatics Center (BBC) at UPRRP, for almost 25 years, we have been developing the “Inverse Probability (InvProb)” theory and its applications in PR. InvProb is usually called (somewhat misleadingly) “Bayesian Statistics”. InvProb is the application of probabilistic logic to make direct inferences to scientific questions. In the talk, we describe some of its contributions and predictions to society, science, and statistical methodology. The warning is that the population dynamics of Puerto Rico, is leading to a dramatic loss and aging of the population. This facts motivates urgent action of public policy. I will recognize several supporters, students, and collaborators in the talk.

## 2 Sesiones Temáticas/ Thematic Sessions

### 2.1 Exploring strategies for Convergent Research

In this thematic session, we will explore convergent research: how various research fields can integrate and share knowledge and expertise to collaborate on complex problems that would be unattainable individually. Participants will gain insights into the methodologies, challenges, and successes of convergent research projects, fostering a deeper understanding of how interdisciplinary collaboration can drive innovation and solve real-world problems. We will engage in discussions based on concrete existing projects, and explore the concept of convergent research, with an emphasis on the potential contribution of Mathematics and Computer Science from the SIDIM community, and their integration with fields such as Biological Science, Education and Engineering.

The workshop will begin with an introduction to the principles and significance of convergent research, setting the stage for a deeper exploration of its applications. Subsequent presentations will delve into (i) the integration of digital resources in teaching statistics, demonstrating how modern technology and the use of real research data can transform and enhance educational approaches; (ii) the development of accessible educational tools, specifically the use of artificial intelligence with digital microscopes and touch tablets to support blind students; and (iii) the co-development of AI tools and experimental biology setups for pollinator monitoring, illustrating the synergy between computer science, engineering and biological research.

**Acknowledgements:** This thematic session is supported by the National Science Foundation, awards CNS-2318597, DRL-23217607.

## **A convergent research strategy to improve STEM Education: a case study for understanding bee pollinator's decline and their impact on food security**

*Jose L. Agosto Rivera*, Biology Department, University of Puerto Rico at Río Piedras.

Convergent research is an innovative approach that integrates knowledge, methods, and expertise from multiple disciplines to address complex scientific and societal challenges. This presentation will provide an introduction to the principles and practices of convergent research, emphasizing its significance in solving intricate problems that cannot be tackled by a single discipline alone.

The focus will then shift to the application of convergent research in monitoring bee pollinator populations and health. Bees play a crucial role in pollination, which is vital for ecosystem stability and agricultural productivity. However, bee populations are facing significant threats from habitat loss, pesticides, diseases, and climate change, and their biological and behavioral response is not yet fully understood.

We will highlight the transformative impact of convergent research on multiple scientific fields, including biology, education, mathematics, computer science and the interdisciplinary initiatives supported such as the NSF Arcibo C3 project. This approach fosters collaboration across these domains, driving advancements in our understanding and conservation of bee pollinators, and providing unique opportunities to innovate in the educational interventions towards students at all levels.

**Acknowledgements:** This work is supported by the National Science Foundation, awards CNS-2318597, DRL-23217607.

## **Transformando la Enseñanza de la Estadística: Integración de Recursos Digitales**

*Jairo A. Ayala-Godoy*, Instituto de Estadística y Sistemas Computarizados de Información, Facultad de Administración de Empresas, University of Puerto Rico at Río Piedras.

*Eugenio Guerrero Ruiz*, Department of Mathematics, University of Puerto Rico at Río Piedras.

La estadística es una disciplina esencial en múltiples campos, donde la toma de decisiones basada en datos es clave. Sin embargo, su enseñanza enfrenta desafíos significativos, como la abstracción de sus conceptos, la dificultad para trasladar la teoría a la práctica y la percepción de la estadística como una materia compleja. Como parte del compromiso de Arcibo C3 con la educación y el fortalecimiento del pensamiento científico en Puerto Rico, esta charla explora cómo la integración de recursos digitales y el uso de datos reales provenientes de investigaciones pueden transformar la enseñanza de la estadística. A través de herramientas

interactivas como CODAP, StatCrunch, QGIS, GeoGebra, GAPMINDER, NetLogo, entre otros, buscamos hacer que la estadística sea más accesible y atractiva para los estudiantes, fomentando la visualización, el análisis y la interpretación de datos en escenarios reales. Nuestro proyecto dentro del marco de Arecibo C3 busca conectar la educación en estadística con datos obtenidos de investigaciones relevantes para la comunidad, permitiendo que el estudiantado desarrolle habilidades analíticas aplicadas a problemas concretos. Al vincular la enseñanza con la exploración de datos del mundo real, promovemos un aprendizaje más profundo y significativo, fortaleciendo el papel de la estadística como herramienta clave para la toma de decisiones informadas en diversas disciplinas.

**Palabras clave:** enseñanza de la estadística, recursos digitales, datos reales, aprendizaje aplicado

## **Integrando la inteligencia artificial con microscopios digitales y una tableta táctil para Estudiantes Ciegos**

Jose M. Alvarez, Facultad de Educación, University of Puerto Rico at Río Piedras.

Esta presentación se discute una innovadora integración de la inteligencia artificial con microscopios digitales y tabletas táctiles, diseñada específicamente para mejorar la experiencia educativa de estudiantes ciegos en el ámbito de las ciencias. La combinación de estas tecnologías permite a los estudiantes ciegos interactuar de manera más efectiva con el material microscópico, proporcionando descripciones auditivas detalladas y retroalimentación táctil en tiempo real. La inteligencia artificial analiza las imágenes capturadas por el microscopio y genera descripciones verbales, mientras que la tableta táctil ofrece una representación táctil de las muestras. Este enfoque no solo facilita el acceso a la información científica, sino que también promueve la inclusión y la participación activa de los estudiantes ciegos en actividades de laboratorio.

**Palabras clave:** Educación inclusiva, Inteligencia Artificial, Estudiantes Ciegos

## **The need for integrating disciplines in AI enhanced animal behavior analysis**

Rémi Mégret, Department of Computer Science, University of Puerto Rico at Río Piedras.

This presentation will show the potential of AI for enhancing the monitoring of animals for biological research and ecology and the challenges requiring the deep integration of several disciplines to achieve the co-design of new approaches and technologies. We will use the understanding of pollinator behavior and impact as an example of a complex problem that

requires convergent contributions from Biology, Ecology, Artificial Intelligence, Engineering, Data Science.

The development of automatized flower-patch assays is an example of such co-design, which requires bee biology and experimental design knowledge from Biologists, development of Machine Learning models for bee tracking and identification using colored paint, with the Engineering of hardware and software systems for real-time monitoring and flower management. Other examples include the co-development of improved cyberinfrastructure and data visualization tools for flexible and large-scale monitoring and analysis in the field or in an educational context, which present new sets of challenges and expertise.

We will explore through these examples the necessary iterative processes of refinement of ideas and experimentation, with the benefit of creating new approaches capable of expanding our understanding of biological systems while strengthening the skills of our students in interdisciplinary innovation.

**Acknowledgements:** Work supported by NSF award CNS-2318597 and USDA NIFA grant 2021-67014-34999.

**Keywords:** AI, pollinator monitoring, multi-disciplinary integration

### 3 Charlas Concurrentes / Concurrent Talks

(In alphabetical order using the first name of the speaker.)

#### **Absolute Irreducibility of Generalized Trinomials Defined by APN Functions of The Form $f(x) = x^i + h(x)$ over $\mathbb{F}_2^s$**

*Alec S. Zabel-Mena*, Department of Mathematics, University of Puerto Rico at Río Piedras.  
*Carlos A. Agrinoni*, Department of Mathematics, Purdue University.  
*Heeralal Janwa*, Department of Mathematics, University of Puerto Rico at Río Piedras.

A multivariate polynomial is said to be absolutely irreducible over a field  $\mathbb{F}$  provided it is irreducible over the algebraic closure of  $\mathbb{F}$ . In algebraic geometry, coding theory, and cryptography, the absolute irreducibility of certain algebraic curves defined by multivariate polynomials is important for determining and solving various problems such as: counting rational points using the Weil conjectures, determining error-correction capabilities, and determining suitability for use in cryptographic systems. One can define such curves using almost perfect non-linear (APN) functions, which are a class of Boolean functions important in coding-theory and cryptography, and find their use as  $S$ -boxes in cryptographic systems.

There exist many methods for determining the absolute irreducibility of an algebraic curve over a field. One such method is testing for irreducibility for sufficiently many extensions of the base field, which is costly to implement algorithmically. One method however does

not depend on testing irreducibility in multiple field extensions. All that is required is that the multivariate polynomial has square-free leading term, and that the terms be coprime. In some cases, we can omit the gcd criteria between terms by using *CCZ*-equivalence between APN functions.

We are interested in when certain algebraic curves defined using APN functions are absolutely irreducible over the finite field  $\mathbb{F}_q$ , where  $q = 2^s$ . We look at the curve defined by:

$$\phi_f(X, Y, Z) = \frac{f(X) + f(Y) + f(Z) + f(X + Y + Z)}{(X + Y)(Y + Z)(X + Z)}$$

as a generalized trinomial of given degree-gap, where  $f(x) = x^i + h(x)$  is a trinomial APN function. We test the absolute irreducibility of  $\phi_f(X, Y, Z)$  over  $\mathbb{F}_q$  on the basis of the square-free criteria, the gcd criteria, and the degree-gap of  $\phi_f(X, Y, Z)$ . We test the absolute irreducibility of  $\phi_f(X, Y, Z)$  for various  $f(x) = x^i + h(x)$ . For those  $i$  in which  $\phi_f(X, Y, Z)$  cannot be tested on the present criteria directly, we present alternate methods for testing absolute irreducibility. In particular when  $i = 24$ , it is sufficient to check for irreducibility in  $\mathbb{F}_{q^7}$ .

**Keywords:** Algebraic Curve, APN Functions, Absolute Irreducibility, Generalized Trinomial, Coding-Theory, Algebraic Geometry

## Automated Detection of Identification Documentation with Open Source Face Detection and Text Recognition libraries

*Anasofía Colón Santiago*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*José Ortiz Ubarri*, Department of Computer Science, University of Puerto Rico at Río Piedras.

The main concern when a system has been compromised is the leak of personal identifying information (PII), particularly official identification documentation (ID), such as driver's licenses and passports. This information can be exposed on the Internet (dark web), used as a ransom, or used to steal the victims' accounts or identities. In some cybersecurity events where IDs have been leaked, the owners of the system did not even know about the presence of such information in their system. In this work, we present an accessible application to automatically detect identifying documentation in file systems with the use of open-source face detection and text recognition libraries. We also describe the work that has been done to increase speed performance and reduce false positives.

**Acknowledgements:** This research sponsored in part by National Science Foundation with CAHSI Local Research Experiences for Undergraduates. Thanks to CSLab and my mentor José Ortiz for their guidance, and to Scholar Compass for providing essential resources.

Grateful for the scholarship supporting my work.

**Keywords:** cybersecurity, file system, detection, identification

## The Actuarial Profession

*Arnaldo Cruet*, SSS de Puerto Rico.

The actuarial profession plays a pivotal role in managing and mitigating risks in various industries. The profession relies heavily on mathematics, statistics, finance, and business to solve various problems. This abstract provides an overview of the basics of the actuarial profession, its key functions, educational requirements, and an overview of the various applications.

Actuaries are highly skilled professionals who use mathematical and statistical methods to analyze the financial consequences of risk and uncertainty. Primarily employed in insurance, pension, and investment sectors, actuaries are instrumental in designing insurance policies, pension plans, and investment strategies that balance risk and financial stability.

Actuaries typically pursue a rigorous course of study that includes mathematics, statistics, economics, and finance. Many actuaries obtain professional credentials, such as those offered by renowned actuarial societies like the Society of Actuaries (SOA) or the Casualty Actuarial Society (CAS), to validate their expertise and enhance their career prospects.

The core functions of actuaries involve assessing risk, determining premium rates, and projecting future financial outcomes. In the insurance sector, actuaries analyze demographic data, health trends, and historical claims to set insurance premiums that ensure the financial viability of insurance companies. In pension planning, actuaries calculate contributions required to meet future pension obligations while considering factors like life expectancy and market conditions. As part of their daily work, actuaries contribute significantly to investment decision-making by evaluating the financial risks associated with different asset classes.

The actuarial profession has expanded beyond traditional domains, finding applications in diverse fields such as healthcare, government, and consulting. Actuaries in healthcare analyze the financial impact of medical treatments, evaluate health insurance programs, and design cost-effective strategies. Employment for actuaries may be found both in the public and private sectors. Government agencies may employ actuaries to assess the financial implications of policy decisions and demographic changes.

In conclusion, the actuarial profession is a dynamic and multidisciplinary field crucial to managing risks and ensuring financial stability across various industries. Actuaries are valued for their strong mathematical foundation and professional credentials. They provide invaluable insights that drive informed decision-making in a dynamic financial landscape. As the world continues to grapple with uncertainties, the role of actuaries remains in high demand in order to mitigate the impact of risks on individuals, businesses and society.



**Keywords:** actuarial, actuaries, SOA, risk management, insurance, financial modeling, data analysis, mathematics, statistics

## Conditional McKean-Vlasov SDEs with Jumps and Regime Switching and Related PDEs

*Carlos Carvajal-Ariza*, Department of Mathematics, University of Puerto Rico at Río Piedras.  
*Son Lu Nguyen*, Department of Mathematics and Systems Engineering, Florida Institute of Technology at Melbourne.

In this presentation, we consider a mean-field stochastic differential equation, also called McKean-Vlasov equation, with initial data  $(t, x, i_0) \in [0, T] \times \mathbb{R}^d \times \mathcal{M}$ , whose coefficients depend on both the solution process  $X_s^{t,x,i_0}$  and its law. By considering square integrable random variables  $\xi$  as initial condition for this equation, we can show the flow property of the solution  $X_s^{t,\xi,i_0}$  of this new equation. Also, we show the well-posedness of the mean-field stochastic differential equation. We extend the work of T.Hao and J. Li (2016)) in a non-trivial manner to mean-field SDEs which, in addition to the driving Brownian motion, are governed by a compensated Poisson random measure and regime switching. We show that under suitable regularity assumptions on the coefficients of the SDE, the solution  $X^{t,x,\mathbb{P}_\xi}$  is twice differentiable with respect to  $x$  and its law, in which the so-called Lion's derivative plays an important role. We establish a new Itô formula, which is a connection to the associated nonlocal integral-PDE, and we show that  $V(t, x, \mathbb{P}_\xi, i_0) = \mathbb{E}[\Phi(X_T^{t,x,\mathbb{P}_\xi,i_0})]$  is the unique classical solution  $V : [0, T] \times \mathbb{R}^d \times \mathcal{P}_2(\mathbb{R}^d) \times \mathcal{M} \rightarrow \mathbb{R}$  of this nonlocal integral-PDE with terminal condition  $\Phi$ .

**Keywords:** Mean-field stochastic differential equation; McKean-Vlasov equation; Lion derivative; PDE of mean-field type

## Application of Adaptive Bayes Factors to Balanced ANOVA Models

*Daiver Vélez Ramos*, Statistical Institute and Computerized Information Systems, University of Puerto Rico at Río Piedras.

Adaptive Bayes factors are constructed from adaptive significance levels and the commonly known minimum Bayes factors (links between p-value and Bayes factors). These minimum Bayes factors are easy to calculate and explain; however, they do not behave as a Bayes factor. For example, they do not change with the size of the sample. Reason why they are adjusted with adaptive significance levels, see Vélez et al. (2022). What they are trying to establish with these adaptive Bayes factors is an equivalence between the common practice

in Frequentist statistical analysis to draw conclusions based on statistical significance and the Bayes factors from the Bayesian point of view. In this talk we present an adaptive Bayes factor proposed by Vélez et al. (2023) to balanced analysis of variance (ANOVA) models in different configurations: one-way ANOVA and two-way ANOVA.

**Keywords:** P-value Calibration; Adaptive Bayes Factor; ANOVA; Adaptive Significance Levels; Linear Models

## 3D Bee Avatar Parametrization and Use in Synthetic Image Data Generation for Computer Vision Application

*Christian J Matos Rivera*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*David Flores*, Department of Computer Science, University of Puerto Rico at Río Piedras.

Deep learning models for computer vision tasks require extensive annotated datasets, yet data scarcity remains a critical challenge, especially for species-specific applications. We refer to synthetic image data as data that can be generated through artificial means, including capturing in virtual environments. Synthetic data generation using 3D avatars and virtual environments offers a promising solution by leveraging advances in graphics rendering to bridge the realism gap. This presentation highlights preliminary steps toward generating synthetic datasets of honeybees, focusing on 3D model preparation, pipeline optimization, and their applicability to detection tasks. By integrating pre-built parametrized bee models with real data reconstructions, we aim to achieve morphological accuracy and rigging fidelity for realistic pose randomization.

Recent work has demonstrated the effectiveness of synthetic data in improving model generalizability, with detection models achieving high mAP scores trained on synthetic datasets. Our pipeline involves refining 3D flexible and parametrized bee models, configuring *Unreal Engine* environments utilizing tools found in the state of the art, and generating annotated images for detection tasks for bees as subjects. This presentation explores the potential of scalable synthetic data generation for ecological monitoring of bees, while addressing trade-offs in model complexity and computational cost.

**Acknowledgements:** This material is in part based upon work supported by the National Science Foundation under Grant No. 2318597.

**Keywords:** computer graphics, 3D models, computer vision, synthetic data, deep learning

## **Simulación numérica del terremoto M6.4 de Puerto Rico 2020: Modelado con mallas no estructuradas y SPECFEM3D**

*Daniel A. Melo Jojoa*, Departamento de Matemáticas, Universidad de Puerto Rico en Mayagüez.

El terremoto de magnitud 6.4 que afectó al sur de Puerto Rico, el 7 de enero de 2020, destacó la necesidad de mejorar los modelos de simulación sísmica en regiones con barimetría, topografía compleja y geología heterogénea. Esta investigación tiene como objetivo desarrollar una malla no estructurada que intente representar de la mejor forma las características geológicas y topográficas de la zona, permitiendo una simulación realista de la propagación de ondas sísmicas. Para ello, se están recopilando datos barimétricos, geológicos y topográficos de fuentes como el Servicio Geológico de los Estados Unidos (USGS) y de Oficina Nacional de Administración Oceánica y Atmosférica (NOAA). Estos datos se procesan utilizando herramientas de Sistemas de Información Geográfica (GIS), como QGIS, para luego generar la malla no estructurada mediante el software GMSH. La malla resultante se utilizará en simulaciones numéricas, usando el software SPECFEM3D, para estudiar la respuesta sísmica del terreno durante el terremoto de 2020. Aunque el proyecto se encuentra en su fase inicial, se espera que los resultados contribuyan a una mejor comprensión de los efectos de los terremotos en Puerto Rico, proporcionando información valiosa para la planificación urbana y la mitigación de riesgos. Este trabajo también sienta las bases para futuras investigaciones que integren modelos más detallados de la corteza terrestre y simulaciones de escenarios sísmicos potenciales. La presentación discutirá los avances actuales, los desafíos encontrados y el plan de trabajo propuesto para completar la investigación.

## **Design of a personalized, gamified virtual experience to strengthen scientific research skills**

*David Flores*, University of Puerto Rico at Río Piedras.

The advancement of scientific research skills is crucial for preparing students to become competitive future scientists. However, many universities face challenges in offering skill-development opportunities, particularly those with limited resources. This situation provides an opportunity to explore innovative developments in natural language processing and machine learning to develop Artificial Intelligence Assistants (AIA), as well as the interactive elements of modern gaming to enhance user experiences in educational contexts.

Recent research has highlighted the significant advancements and increasing integration of AIA within educational settings. Despite these promising developments, many interactions between users and AIA tend to feel impersonal. This lack of personalization often neglects individual user preferences and needs, resulting in decreased usage and, in some cases, the eventual abandonment of these technological tools over time.

Gamification refers to the strategic application of game-like elements and principles in

non-game contexts to influence and enhance user behavior. This approach leverages mechanisms—such as point systems, challenges, and rewards—found in games to create engaging experiences in settings, such as education, marketing, and health. By integrating these elements, gamification aims to foster user participation and motivation, making the activities more enjoyable and compelling. Recent studies highlight that incorporating AIA allows for further personalization of these motivational strategies, enabling a tailored experience that resonates with individual user preferences and enhances user engagement even more effectively.

In this presentation, we will discuss the development of a comprehensive blueprint for an engaging and interactive virtual experience. Specifically, we will focus on the creation of an initial prototype that provides tailored and guided practices designed to enhance the scientific research skills of undergraduate Hispanic students. By incorporating gamification elements, the aim is to foster a dynamic learning environment that not only motivates students but also equips them with essential competencies for success in their academic and professional careers in the sciences.

**Acknowledgements:** This is joint work with Elizabeth Dvorsky, Joseph Carroll-Miranda, Carlos Vázquez-Echevarría, and Edusmildo Orozco, all from the University of Puerto Rico at Río Piedras. This project is supported by the National Science Foundation under Award # 2421165. Carlos Vázquez-Echevarría is also partially supported by the PEAFF program of the University of Puerto Rico at Río Piedras.

**Keywords:** Gamification, virtual intelligent agent, research skills

## **Accessing the probability of the next Hurricane Maria in 25 years and mortality assesment**

*David Torres*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Luis R. Pericchi Guerra*, Department of Mathematics, University of Puerto Rico at Río Piedras.

Non-Bayesian methods are known to systematically underestimate (usually by a large margin) the probability of extremes (disasters). We present initial results on a comprehensive Bayesian framework of the probability of extremes (floods or drought) and the potential “excess mortality” associated using extreme value distribution estimation with Bayesian Hierarchical Modeling. Furthermore, this framework will allow us to estimate the excess net emigration, incorporating conditioning to extreme events such as major hurricanes such as Maria. Hurricane Maria struck Puerto Rico on 20 September 2017, marked by significant controversy and debate. This is particularly true with respect to the excess mortality associated with the disaster. (Kishore et al., 2018; Rios et al., 2021; Santos-Burgoa et al., 2018; Rivera & Rolke, 2018).

A comprehensive Bayesian framework for estimating the probability of extreme events

such as floods or droughts and their potential "excess mortality." Firstly, we estimate the probability parameters of another extreme event, such as Hurricane Maria impacting Puerto Rico. We analyze the daily maximum precipitation from NOAA, Coloso, Aguada, and Puerto Rico stations using Bayesian MCMC Generalized Extreme Value. The maximum rainfall at Coloso Aguada rain gauge during Hurricane Maria was 10 inches daily. We used Bayesian Hierarchical modeling to calculate the posterior probability of extreme rainfall associated with Major Hurricane Maria. Finally, the estimates of the likelihood of getting an event equivalent to the recorded extreme rain are reported. The posterior probability calculations to observe a major rainfall, as observed during Hurricane Maria in 2017, were close to 1%. The return level for 25 years was calculated with the Generalized Extreme values reported as 10.0 daily inches of rain. Quantiles of the MCMC sample from the posterior distribution with a posterior mean 100-year level of 10 (7.27, 15.42) inches. The probability of getting another H. Maria shortly (i.e., 25 years) is not neglected and is reported as close to 1%.

## **OiPR: Pensamiento Computacional, Inteligencia Artificial y Programación Competitiva en Puerto Rico**

*Edwin Flórez*, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

Las Olimpiadas Informáticas de Puerto Rico (OiPR) representan una iniciativa innovadora orientada a fortalecer la educación en informática, pensamiento computacional, inteligencia artificial y programación competitiva entre estudiantes preuniversitarios en Puerto Rico. Inspirada en iniciativas internacionales como la Bebras Challenge, la International AI Olympiad (IAIO) y la International Olympiad in Informatics (IOI), OiPR busca reducir la brecha en la educación en computación y preparar a jóvenes talentos para competir en escenarios globales.

En esta charla, se presentará el origen, la estructura y la evolución de OiPR, resaltando su impacto en estudiantes y educadores. Se discutirán los desafíos y oportunidades asociados con la implementación de una competencia nacional, tanto en informática como en inteligencia artificial, así como estrategias para la formación docente y la motivación estudiantil. Además, se compartirá la experiencia de la primera edición de la olimpiada nacional de informática y la participación de Puerto Rico en competiciones internacionales como la Olimpiada Iberoamericana de Informática (OII) y la International Artificial Intelligence Olympiad (IAIO), con una proyección hacia futuras participaciones en otras competiciones internacionales.

Esta iniciativa no solo promueve el desarrollo del talento local en informática y programación, sino que también abre la puerta a colaboraciones interdisciplinarias con las comunidades de matemáticas, ciencias de la computación y otras disciplinas científicas, contribuyendo así al fortalecimiento de la enseñanza de la computación en Puerto Rico.

**Palabras clave:** Inteligencia artificial, pensamiento computacional, resolución de problemas, educación en informática, desafíos de pensamiento computacional, programación competitiva, competencias internacionales, algoritmos y lógica, habilidades digitales

# Modeling nectar dynamics and integration into a robotic system for autonomous data collection in honey bees

Edwin R. Lara-Perez, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The study of honeybee feeding behavior is crucial in neurobiology, ecology, and evolution. A useful approach involves the use of artificial feeders with specific signals to analyze their behavior. Researchers at the University of Puerto Rico in Río Piedras previously used these feeders by manually filling them and collecting data, which required long working hours and an inefficient process.

To optimize this procedure, an automated system was developed based on a 555 oscillator circuit in an astable configuration, where the output frequency correlates with the volume of nectar present, allowing the detection of quantities as small as 4  $\mu\text{L}$ . The validation process included field experiments and controlled environment tests, where it was identified that nectar measurements varied with environmental conditions. To address this variability, data was collected using an Arduino, and an exponential regression model was implemented to estimate the state of available nectar in the flower over time.

In this talk, we will present the Arduino-based automated system, the mathematical model, and the prototype design of an artificial flower patch, aiming to improve data collection efficiency and the reliability of research results. Finally, we will discuss the potential integration of a more realistic model of nectar dynamics to describe the evolution of nectar concentration and volume under dynamic environmental conditions, contributing to the automation and optimization of pollinator studies.

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**Keywords:** Data collection, modeling, automation, optimization, Arduino, 555 oscillator, nectar detection

# Linear Recurrences associated to k-Rotation Symmetric Boolean Functions

Eiver Rodríguez-Pérez, Department of Mathematics, University of Puerto Rico at Río Piedras.

Exponential sums of symmetric Boolean functions are linear recurrent with integer coefficients. The concept of k-trapezoid Boolean function is also introduced and it is showed that the linear recurrences that exponential sums of k-trapezoid Boolean functions satisfy are the same as the ones satisfied by exponential sums of the corresponding k-rotations symmetric Boolean functions for any sequence of functions in the form  $S(k-R_{1,2,\dots,s-1,s}(kj))$ . Finally, it is proved that exponential sums of trapezoid and symmetric polynomials also satisfy linear recurrences with integer coefficients over any Galois field  $\mathbb{F}_q$ .

**Acknowledgements:** This is joint work with José E. Calderón-Gómez (University of Puerto Rico at Mayagüez), Luis A. Medina (University of Puerto Rico at Río Piedras), and Carlos A. Molina-Salazar (University of Puerto Rico at Arecibo).

**Keywords:** Boolean Functions, Exponential Sum, k-Trapezoid, Linear Recurrences

## Impacto y resultados de la plataforma WebWork como herramienta complementaria en la enseñanza de cursos de matemáticas y estadística en estudiantes de la Universidad Militar Nueva Granada

Eliseo Gallo Albarracín, Departamento de matemáticas, Universidad Militar Nueva Granada, Colombia.

Isnardo Arenas Navarro, Departamento de matemáticas, Universidad Militar Nueva Granada, Colombia.

Edwin Flórez Gómez, Departamento de Matemáticas, Universidad de Puerto Rico en Mayagüez.

El término de Evaluación Asistida por Computadora (Computer Assessment Instruction en inglés) se refiere al uso de evaluaciones basadas en computadora en el contexto educativo, lo que implica la creación y administración de pruebas y tareas a través de plataformas digitales. Estas evaluaciones se realizan electrónicamente, permitiendo elementos interactivos y retroalimentación inmediata para los estudiantes. Los datos generados por estas evaluaciones se analizan para informar estrategias de enseñanza, y la flexibilidad en cuanto al momento y la accesibilidad aseguran la inclusión. Esta charla pretende presentar los resultados obtenidos desde la percepción de algunos estudiantes de la Universidad Militar Nueva Granada al utilizar sistemas web diseñados para mejorar sus habilidades en matemáticas, mediante el uso de una plataforma WebWork. La plataforma WebWork implementa procesos de generación de preguntas dinámicas aleatorias en temas contenidos en algunos cursos ofrecidos en la Universidad Militar Nueva Granada. En este contexto, se pretendía identificar

cómo la aleatoriedad y la dinámica en la generación de preguntas pueden influir en la percepción y el desempeño de los estudiantes. Se realizó un estudio experimental en donde fue seleccionada una muestra de estudiantes, que se encuentran cursando clases de matemáticas o estadística. Se recopiló información cuantitativa a través de cuestionarios de percepción y se registraron datos sobre el rendimiento de los estudiantes en las tareas. Los resultados del estudio indican que el uso de una plataforma de tareas con generación de preguntas dinámicas aleatorias, tuvo un impacto positivo en la percepción de los estudiantes hacia el aprendizaje de matemáticas. En general, se pudo observar en los estudiantes una mayor satisfacción y motivación al estudiar matemáticas a través de esta plataforma en comparación con métodos de estudio tradicionales.

**Agradecimientos:** Estos resultados se presentan como resultados del proyecto de investigación CIAS-3793, patrocinado por la Universidad Militar Nueva Granada, Colombia.

**Palabras clave:** Evaluación Asistida por Computador, WebWork, percepción estudiantil

## **Modelo para la enseñanza de la resolución colaborativa de problemas matemáticos: Aprendizajes de una investigación**

*Omar Hernández Rodríguez*, Departamento de Estudios Graduados, Universidad de Puerto Rico en Río Piedras.

*Eric Figueroa González*, Escuela Elemental, Universidad de Puerto Rico en Río Piedras.

*Yamily Colón Negrón*, Escuela Elemental, Universidad de Puerto Rico en Río Piedras.

Reportamos un estudio de caso sobre la resolución colaborativa de problemas matemáticos en nivel elemental. En el estudio participaron estudiantes de quinto grado organizados en subgrupos de dos o tres estudiantes. La investigación tenía tres fases. En la primera los estudiantes resolvieron problemas sin ninguna instrucción ni supervisión del proceso, en la segunda, recibieron instrucción sobre el proceso colaborativo con énfasis en la comprensión común del problema y tuvieron supervisión y, en la tercera, volvieron a resolver problemas sin instrucción, ni supervisión. En cada fase los estudiantes resolvieron 3 problemas a razón de uno por periodo de clases. Las hojas de trabajo de los alumnos, junto con las grabaciones en video y entrevistas semi estructuradas constituyeron las fuentes de información del estudio. Utilizando el modelo establecido por la OECD (2017), el análisis de la información permitió establecer un modelo para la enseñanza colaborativa de problemas matemáticos basado en seis categorías indispensables. Las seis categorías son (1) propiciar el ambiente para que el estudiante adquiera compromiso en la resolución colaborativa del problema, (2) posibilitar la comprensión individual y grupal del enunciado del problema, (3) estimular la generación y prueba de posibles soluciones, (4) mantener el compromiso en la resolución colaborativa del problema, (5) mantener equilibrio entre trabajo colaborativo e individual, y (6) verificar la comprensión individual y grupal sobre el problema.



# Multivariate Mittag-Leffler Solution for a Forced Fractional-Order Harmonic Oscillator

*Eugenio Guerrero Ruiz*, Department of Mathematics, University of Puerto Rico at Río Piedras.

*Jessica Mendiola-Fuentes*, Department of Basic Sciences and Engineering, University of the Caribbean, Mexico.

*Juan Rosales-García*, Department of Electrical Engineering, Irapuato-Salamanca Campus, University of Guanajuato, Mexico.

The harmonic oscillator is a cornerstone physical and mathematical system that underpins the modeling of diverse phenomena across various domains of physics. By incorporating fractional derivatives in place of classical derivatives, the system's behavior can be extended to encompass a broader spectrum of dynamical responses. Introducing fractional derivatives to each term in the governing differential equation significantly increases the complexity of deriving and analyzing analytical solutions for such systems.

In this talk, we present solutions to the multi-term forced fractional harmonic oscillator, formulated as

$$D^{2\gamma}x(t) + AD^\gamma x(t) + \omega_0^2 x(t) = F(t),$$

where  $D^\gamma x(t)$  denotes the Caputo fractional derivative of order  $\gamma$  of the function  $x(\cdot)$ , subject to the initial conditions  $D^0 x(0^+) = 0$  and  $D^1 x(0^+) = b_1 \in \mathbb{R}$ , for  $\gamma \in [0.5, 1]$ . Here,  $A$  represents the damping coefficient,  $\omega_0^2$  corresponds to the spring-mass restoring force, and  $F(\cdot)$  denotes the external driving force, typically oscillatory in nature. Using the Laplace transform technique, we derive analytical solutions expressed in terms of multivariate Mittag-Leffler functions.

Additionally, we examine the dynamics of both damped and undamped free fractional harmonic oscillators. Numerical simulations illustrate the impact of fractional derivatives on system dynamics and validate the consistency of the model by recovering classical integer-order results as a limiting case. These findings highlight the potential of fractional-order modeling to capture intricate physical phenomena with greater fidelity.

**Keywords:** fractional forced oscillator; Mittag-Leffler function; fractional calculus; Laplace transform

## Model Prior Probabilities: Protecting against overfitting and multiple comparisons

*Fernando E. Betancourt Vélez*, Department of Mathematics, University of Puerto Rico at Río Piedras and University of Puerto Rico at Cayey.

*Luis R. Pericchi Guerra*, Department of Mathematics, University of Puerto Rico at Río Piedras.

Objectively assigning prior probabilities to models is crucial for inference in Bayesian model selection, influencing posterior probabilities and model selection criteria. This presentation explores various approaches to specifying model prior probabilities, including constant, Jeffreys, Scott and Berger prior, and the Harmonic Weights prior. The latter, originally proposed by Berger and Pericchi (1996) but first studied here, balances parsimony and multiple comparisons through a decreasing weight structure. Additionally, we discuss the role of these priors in Bayesian Model Averaging, where model uncertainty is integrated into inference.

**Keywords:** Model Selection, Bayesian Statistics or Inverse Probability

## Determining the parameters of orthogonal Grassmann codes

*Fernando L. Piñero*, Department of Mathematics, University of Puerto Rico at Ponce.

The polar orthogonal Grassmann code  $C(\mathbb{O}_{3,6})$  is the linear code associated to Plücker's map of the polar orthogonal Grassmannian subvariety. With elementary techniques we prove that the minimum distance of the polar orthogonal Grassmann code is  $q^3 - q^2$  for  $q$  odd and  $q^3$  for  $q$  even. Our technique is based on partitioning the orthogonal space into different sets such that on each partition the code is identified with evaluations of determinants of skew-symmetric matrices. Our bounds come from elementary algebraic methods counting the zeroes of particular classes of polynomials. Our elementary techniques can be adapted to the  $C(\mathbb{O}_{4,8})$  case.

**Acknowledgements:** This is joint work with Sarah Gregory (University of Richmond), Doel Rivera-Laboy (University of Kentucky), and Lani Southern (Willamette University).

**Keywords:** Grassmannian, polar spaces, finite fields

# Forecasting and optimizing network operations using machine learning for temporary IT infrastructure in high-demand scenarios

*Freddy Bello*, Department of Computer Science, University of Puerto Rico at Río Piedras.  
*Edusmildo Orozco*, Department of Computer Science, University of Puerto Rico at Río Piedras.

This research explores leveraging machine learning to enhance network operations for short-term, high-demand IT infrastructures such as those deployed at trade shows or high-density events. Current network forecasting approaches are reactive, relying heavily on manual intervention. However, temporary IT setups experience rapid resource utilization fluctuations, which suggests predictive models for optimal performance could be a better fit.

Predictive analytics in temporary IT infrastructures has the potential to enhance efficiency, inform best practices for long-term setups, and ensure a high-quality user experience. Challenges include the short duration of these setups and the need for rapid deployment and forecasting.

In this work we aim at applying machine learning techniques such as Long Short-Term Memory (LSTM), Autoregressive Integrated Moving Average (ARIMA), and Gradient Boosting to analyze short-lived telemetry datasets, to enable more accurate forecasting and proactive resource management. For this presentation we will showcase the viability of our approach to produce a high-confident forecast by using a modular regression model over the current dataset.

**Acknowledgements:** Data source for this research has been collected during CISCO Live events from CISCO Systems.

**Keywords:** network design, machine learning, ARIMA, LSTM, trade show

## Phenomenological Quantum Resurgence

*Gabriel A. Coloma Irizarry*, Department of Mathematics, The Ohio State University, Columbus.

In a long series of papers, Costin et al. analysed variations of 1d quantum systems with one-center Hamiltonian point-interactions. In particular, they study associated Floquet problems with parametric perturbing potentials that model e.g. photoemission effects via radiation from external oscillating fields. Their methods rely on global analysis of analytic recurrences of the Laplace transformed problem and are valid in nonperturbative regimes for all values of the amplitudes and frequencies, of importance with the advent of high-intensity laser technology in modern optics. In our setup, we illustrate the case of multi-frequency-interference and construct convergent transseries expansions that capture long-time asymptotics of ionization

amplitudes and other phenomenological consequences like multi-photon processes and Fermi Golden Rule exponents.

## Introduction to the Mathematical Theory of Knots

Gabriel Montoya, Department of Mathematics, University of Puerto Rico at Río Piedras.

The fascinating mathematical theory of knots studies the embeddings of simple closed curves, up to natural deformations, in the three dimensional space. Historically originated as a sub-field of topology, the theory underwent significant scientific advancements so that knot theory is currently a research field of its own. Nonetheless, the theory enjoys broad connections with other fields such as chemistry, biology, physics, and statistical mechanics.

The theory of invariants of knots and links was revolutionized with the discovery of the Jones polynomial in 1984. At the end of the XX century, Mikhail Khovanov announced a novel construction of a new and very powerful link invariant: a homology theory categorifying the Jones polynomial, containing more information, and having a richer algebraic structure. In this talk we briefly discuss the historical origins of the theory and introduce some polynomials. In particular, we focus on the Kauffman bracket polynomial, which was introduced in 1985 and provides a simpler approach to the Jones polynomial. Furthermore, we discuss how Khovanov homology can be constructed directly from the bracket polynomial.

## Developing a Web-Based Software to Guarantee Remote Data Access

Gabriel O. Romero Torres, Department of Computer Science, University of Puerto Rico at Río Piedras.

A shared problem in resource-constrained institutions is having secure remote access to data generated by scientific instrumentation whenever an emergency occurs, such as natural disasters, pandemics, and in the case of Puerto Rico, faults in critical infrastructure. In this work, we present a web-based software system called Molenium to provide secure access to the data generated by scientific instrumentation. The system securely synchronizes the data generated by the instruments with a remote server accessible from anywhere through a web interface. We will discuss how we automate the configuration of the scientific instruments to synchronize with the server securely and how the data is presented to the users with different access levels.

**Acknowledgements:** This is joint work with Adriana P. Rivera Sánchez, Gabriel O. Romero Torres, Sebastián A. Ramírez González, and José R. Ortiz Ubarri, all from the

University of Puerto Rico at Río Piedras.

## Cosine-type resolvent families and bounded perturbations

Henry J. Cortez, University of Puerto Rico at Arecibo.

Valentin Keyantuo, Department of Mathematics, University of Puerto Rico at Río Piedras.

We study resolvent families  $(C_\alpha(t))_{t>0}$  on a Banach space  $X$ , associated with the Riemann-Liouville fractional derivative of order  $\alpha \in (1, 2)$ . In particular, we prove the following perturbation result: if  $A$  is the generator of a cosine-type resolvent family  $(C_\alpha(t))_{t>0}$  with  $\alpha \in (1, 2)$  and  $P$  is a bounded linear operator, then  $A + P$  is the generator of a cosine-type resolvent family..

**Keywords:** Riemann-Liouville fractional derivative, Banach spaces, resolvent families, bounded perturbations

## In Silico Analysis of Missense Mutation Impacts on Local Protein Structures

Isabel T. Rivera Plata, Department of Computer Science, University of Puerto Rico at Río Piedras.

David Flores Granados, Department of Computer Science, University of Puerto Rico at Río Piedras.

Proteins adopt three-dimensional structures to perform various functions within the cell. Given that the amino acid sequence shapes the protein's structure, which then dictates its function, mutations at the nucleotide level in the coding region of the genome can have an impact on the translated protein by change of an amino acid. In the context of cancer, mutations recurrently found in tumors are called hotspots. Few hotspots and their role in protein structure alteration have been studied. Recent advances in bioinformatics include the development of machine learning tools for the analysis of effects of genomic mutations on proteins. However, most of these methods are traditionally trained on sequence-based datasets of genes and proteins, which may present a limit in the analysis of these alterations at the structural level.

Mathematical methods such as graph theory can be effective in summarizing and predicting biological characteristics in proteins by framing its structure as networks – objects composed of nodes and edges. In proteomics, amino acids or atoms can play the role of nodes, while bonds and other properties can be represented by edges. Complex, nonlinear relationships within graph-structured datasets can be analyzed through the application of deep learning models such as a graph neural network (GNN), specifically designed to leverage

non-Euclidean data and create node embeddings for further processing. A manually curated dataset containing experimentally determined energy measurements of protein mutations quantifying the impact on stability can serve as input for the model. The corresponding Protein Data Bank (PDB) files can be parsed to extract additional detailed atomic and structural information, including residue coordinates and bonding interactions. GNNs can be trained to identify and cluster significant regions of the protein structure that contribute disproportionately to destabilization. This integrative approach not only provides insight into the structural determinants of protein stability but also lays the foundation for predictive modeling of mutational effects, which has implications for protein engineering and therapeutic design.

**Keywords:** proteins, mutations, three-dimensional structure, graph neural network

## **Remezcla: Una estrategia innovadora para fortalecer la educación informal de ciencias de cómputos a través de pedagogías culturalmente relevantes**

*Isaris R. Quiñones Pérez*, Proyecto Remezcla, UPR - Río Piedras.

El Proyecto Remezcla es una investigación colaborativa entre la Universidad de Puerto Rico, Recinto de Río Piedras (UPR-RP) y el Instituto de Tecnología de Georgia (Georgia Tech) que busca aumentar las destrezas de pensamiento computacional de los estudiantes latinos e hispanohablantes en Puerto Rico y los Estados Unidos. En esta presentación vamos a exponer cómo el proyecto Remezcla ejemplifica el impacto transformador de las pedagogías culturalmente relevantes en la educación informal de la ciencia de cómputos (CS). Remezcla utiliza los activos culturales de estudiantes hispanohablantes, como sus tradiciones y gustos musicales, para atraerlos hacia el aprendizaje de CS. Para muchos de estos estudiantes, las actividades extracurriculares y campamentos de verano organizados por Remezcla son su primera experiencia programando; llegan motivados por la música y se van con un conocimiento básico de programación que les da confianza para seguir explorando el campo.

El proyecto enfrentó retos significativos iniciales, como dificultades con la retención de estudiantes y su renuencia a utilizar conceptos computacionales más allá de su uso utilitario para crear piezas sencillas de música. Estas dificultades se abordaron mediante estrategias como la introducción de actividades dinámicas y culturalmente relevantes, estructurar el aprendizaje en metas claras y alcanzables acompañadas de reconocimiento al logro (en forma de ‘badges), y la integración de actividades sin tecnología (unplugged activities). Estas estrategias no solo mejoraron la retención, sino que también incrementaron el sentido de pertenencia y la autoeficacia de los estudiantes, según los datos recopilados en encuestas posteriores a las actividades.

Además, el proyecto requirió una constante adaptación de nuestra definición de lo que significa ser culturalmente relevante, reconociendo que no se trata solo de incluir elementos culturales musicales, sino de conectar profundamente con las experiencias y necesidades de

los estudiantes. Al relatar nuestras experiencias, retos y aciertos implementando las actividades de Remezcla, queremos que otros educadores vean en la educación informal una ruta accesible hacia la adquisición de destrezas de pensamiento computacional para sus estudiantes.

**Reconocimientos:** Este trabajo fue realizado en colaboración con Pascua Padró Collazo (Proyecto Remezcla, UPR - Río Piedras), Rafael A. Arce Nazario (UPR - Río Piedras), Lilliana M. Marrero Solís (UPR - Río Piedras), y Joseph E. Carroll Miranda (UPR - Río Piedras). El proyecto Remezcla está subvencionado por la Fundación Nacional de las Ciencias (AISL #2005818)

**Palabras clave:** pedagogías culturalmente relevantes, educación informal, ciencia de cómputos

## Khovanov Homology of the $P(\pm 1, m, n)$ pretzel links

*Iván Cardona*, Department of Mathematics, University of Puerto Rico at Río Piedras.  
*Gabriel Montoya-Vega*, Department of Mathematics, The Graduate Center CUNY, NY, and  
 Department of Mathematics, University of Puerto Rico at Río Piedras.

In (2000) Mikhail Khovanov defined, for a given diagram of an oriented link  $\vec{L}$ , a collection of groups  $\mathcal{H}^{i,j}(\vec{L})$  numerated by pairs of integers. These Khovanov groups were constructed as cohomology groups of certain chain complexes, where it turned out that the Euler characteristic of these complexes gave a version of the Jones polynomial of the link. Subsequently, Oleg Viro in (2004), reformulated, for non-oriented framed links, Khovanov’s definitions and obtained “friendlier” Khovanov homology groups  $H_{a,b}(L)$ , where  $L$  is a non-oriented framed link. The classical Khovanov cohomology  $\mathcal{H}^{i,j}(\vec{L})$ , and the framed version of Khovanov homology  $H_{a,b}(L)$ , are related by the following:

$$\mathcal{H}^{i,j}(\vec{L}) \cong H_{w-2i, 3w-2j}(L) \cong H_{a,b}(L) \cong \mathcal{H}^{\frac{w-a}{2}, \frac{3w-b}{2}}(\vec{L}),$$

where  $w(\vec{L}) = w$  is the writhe of the oriented link diagram  $\vec{L}$ .

We use Viro’s approach to compute  $H_{a,b}(L)$  for the pretzel links of the type  $L = P(1, m, n)$  and  $L = P(-1, m, n)$ . The computation involves using the long exact sequence of Khovanov homology and the Khovanov homology groups of torus links of type  $T(2, n)$  which were computed in Montoya-Vega (2023).

**Keywords:** Jones polynomial, Khovanov homology, framed links

## **Ortomosaicos geométricos de alta precisión creados drones para creación y monitoreo de polinizadores con énfasis en la abeja *Apis mellifera***

*Jaime W. Abreu Ramos*, Escuela Secundaria y Facultad Eugenio María de Hostos, Universidad de Puerto Rico en Río Piedras.

Los avances en la tecnología de drones y los software de código abierto han transformado la manera en que mapeamos y monitoreamos la biodiversidad, permitiendo un análisis más preciso y detallado de los ecosistemas. En esta presentación, expondremos los detalles geométricos del del Drone Mapping para la creación de ortomosaicos geométricos de alta precisión, con un enfoque particular en el monitoreo de polinizadores, especialmente la abeja *Apis mellifera*, una especie clave para la salud ambiental y la seguridad alimentaria.

A través del uso de herramientas avanzadas de mapeo, es posible capturar imágenes de alta resolución y convertirlas en modelos georreferenciados que facilitan la identificación de hábitats críticos para los polinizadores. Utilizando sensores que operan en distintas longitudes de onda, como luz visible, térmica y multiespectral, se están desarrollando métodos innovadores para detectar panales silvestres y establecer un sistema de monitoreo a largo plazo. Estos datos son esenciales para comprender la dinámica de las poblaciones de abejas y su interacción con el entorno, lo que permite diseñar estrategias de conservación más efectivas.

La presentación será un espacio de diálogo que explique la matemática inherente al mapeo y en especial sobre la importancia de los ortomosaicos geométricos de alta precisión y su impacto en la investigación científica y la gestión ambiental. Dirigida a académicos, profesionales y con énfasis en la matemática aplicada, la ciencia de datos y la teledetección, esta presentación invita a descubrir cómo los drones y el software libre están revolucionando la cartografía ambiental y abriendo nuevas posibilidades para el estudio y protección de los polinizadores en su hábitat natural.

## **The rich geometry of lines on hypersurfaces and some generalizations**

*Jaziel Torres Fuentes*, Department of Mathematics, University of Notre Dame.

*Eric Riedl*, Department of Mathematics, University of Notre Dame.

Hypersurfaces are one the easiest objects to define in algebraic geometry. They are the first example we all learn of what an algebraic variety is. However, appearances may be deceiving: a lot is unknown about their geometry. After discussing all the required definitions (in simple and intuitive terms), this talk will survey some of the things we know about the family of lines contained in a hypersurface. We will go from the famous “27 lines on a cubic surface” to the Debarre-de Jong conjecture: on a smooth hypersurface  $X$  of degree  $d$  and dimension



$n$ , the dimension of the space of lines on  $X$  should be the “expected” dimension, as long as  $d \leq n + 1$ . This will lead to the study of hypersurfaces of low degree ( $d$  small compared to  $n$ ), for which the Debarre-de Jong conjecture is proven true. On the other hand, sufficiently random hypersurfaces should look the same locally everywhere. In particular, the family of lines passing through an arbitrary point on the hypersurface should be the same, independent of the point chosen, right? Then we can ask: how does the story change if we consider higher dimensional linear subspaces instead of lines? We will answer these questions and a little more. No knowledge in algebraic geometry will be assumed.

**Keywords:** algebraic geometry, hypersurface, linear subspace, dimension

## Analyzing economic recession probabilities through interest rates and labor market indicators

Jeremis N. Morales Morales, Inter American University, San German, Puerto Rico.

Puerto Rico relies heavily on imported resources, such as food and energy, to maintain our standard of living. However, recent economic disruptions, including the 2008 Global Financial Crisis and the 2020 COVID-19 pandemic, have caused a permanent deviation from the steady real GDP growth trend established since the 1950s, leading to unprecedented disruptions. To assess these effects, the United States Treasury bonds and the labor market were analyzed as economic recession indicators, given that bonds reflect demand for safety and liquidity, while the labor market conditions provide insight into current macro-economic conditions. This study introduces new probabilistic models, utilizing Logistic Regression and Random Forest Classifier, to predict the probability of an economic recession. Feature engineering was employed to enhance the models’ performance. Results align with current conditions, highlighting the need for further research on Puerto Rico’s unique economic challenges stemming from its dependence on the United States.

## A Topological Proof of the Fundamental Theorem of Arithmetic

Jhixon Macías, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The purpose of this talk is to present a topological proof of the well-known *Fundamental Theorem of Arithmetic*. As far as we know, this is the first proof of this kind to be introduced. To achieve this, we endow the set of natural numbers with the well-known *divisor topology*. We then demonstrate that if the set of prime numbers is dense in the natural numbers with the divisor topology, the Fundamental Theorem of Arithmetic holds. Finally, using a topological argument, we prove that the set of prime numbers is indeed dense in the

set of natural numbers with the divisor topology.

**Keywords:** divisor topology, fundamental theorem of arithmetic

## **Dynamics of AMR beyond a single bacterial strain: revealing the existence of multiple equilibria and immune system-dependent transitions**

*Jhoana P. Romero-Leiton*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

In this presentation, we explore the growing threat of antimicrobial resistance (AMR), a major global public health issue that complicates the elimination of harmful microorganisms in the host. Mathematical models have significantly contributed to the understanding of AMR dynamics and identifying strategies to combat bacterial infections, although they have primarily focused on single bacterial strains rather than microbial communities. However, research on microbial consortia remains limited.

This study is a step in examining how resistance spreads within microbial communities, emphasizing the ecological dynamics of bacterial competition, and the role of the host immune system in infection control. We introduce a mathematical model that captures AMR propagation by considering competition between two bacterial strains of the same species. Our analysis investigates the stability and bifurcations using different parameters to represent how effectively the host immune system eliminates bacteria.

Our findings indicate that bacterial replication rate and immune system efficiency are key factors in the spread of AMR. Bacteria with lower replication rates can be effectively controlled, leading to disease eradication, whereas those with higher replication rates require a stronger immune response for clearance. The model identified nine biologically feasible equilibrium points, four of which corresponded to the distinct immune system types described in the literature. This highlights the importance of maintaining a strong immune system and adopting responsible antibiotics to slow the emergence and dissemination of AMR.

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**Keywords:** Antimicrobial resistance, mathematical model, Bacterial strains, Immune system, Escherichia coli

## **Nonlinear Dynamics in Financial Models: Traveling Waves in Black-Scholes Equations**

Joaquín Rivera, Department of Mathematics, University of Puerto Rico at Humacao.

The classical Black-Scholes equation, while foundational in financial mathematics, relies on linear assumptions that often fail to capture the complexities of real-world markets. In this talk, we explore a nonlinear extension of the Black-Scholes model by incorporating a cubic nonlinearity, which enables the modeling of phenomena such as feedback effects, saturation, and speculative bubbles. Using a traveling wave ansatz, we reduce the nonlinear partial differential equation (PDE) to a second-order ordinary differential equation (ODE) and analyze the system through phase-plane analysis and stability theory. This approach allows us to derive conditions for the existence of traveling wave solutions, which represent self-propagating patterns in financial markets, such as the spread of market shocks or trends.

We discuss the financial implications of these solutions, highlighting their relevance for investment strategies, risk management, and understanding market behavior. Numerical simulations of both the ODE and the full PDE validate our theoretical results and provide visual insights into the evolution of traveling wave solutions. This presentation bridges mathematical modelling and financial interpretation, offering a fresh lens to understand nonlinear effects in markets.

## **Bayesian methodological innovations for Age-Period-Cohort models**

Jomarie Jiménez González, Department of Biostatistics and Epidemiology, University of Puerto Rico, Medical Sciences Campus.

Bayesian methodologies for Age-Period-Cohort (APC) modeling have been gaining momentum. Full uncertainty of parameters, its current computational viability and its ability to make accurate predictions even for complex phenomena are some of the advantages that favor a Bayesian framework over frequentist techniques. However, implementing Bayesian techniques continues to present some challenges, as they depend on the nature of the problem, which involves considering expert knowledge. In response to this problem, we discuss in this talk our methodological contributions that take into account some of the issues that arise when developing APC models. As a case study, we analyze fertility data for women in Puerto Rico.

We use an APC linear model for fertility with a Poisson likelihood for the event  $y_{a,t}$ , the

number of births that correspond to women in age group  $a$  in period  $t$ . For modeling scales, the use of Scaled Beta2 (SBeta2) distributions is encouraged, instead of the inconvenient Inverted-Gamma/Gamma family heavily used in literature. The advantages of the Scaled Beta2 distribution include its robustness, flexibility, and its reciprocity property. Another contribution is the use of an alternative approach to address the identification problem, which is crucial for separating period and cohort effects. The comparison criteria for both current and novel models are calculated, as it is necessary to determine the weight of the period and cohort effects. APC parameters are assigned Random Walk 2 priors, and several considerations for imposing constraints. In particular, it is important to allow uncertainty instead of setting specific parameters to strictly zero, in order to obtain reasonable probability intervals. Some available software for modeling Age-Period-Cohort events is briefly reviewed.

**Acknowledgements:** This is joint work with Angélica M. Rosario Santos (University of Puerto Rico, Medical Sciences Campus), Luis R. Pericchi Guerra (University of Puerto Rico at Río Piedras), Hernando Mattei (University of Puerto Rico, Medical Sciences Campus), and David A. Torres Nuñez (University of Puerto Rico at Río Piedras). This research is supported by *Estudio Demográfico de Puerto Rico 2024: Análisis y Recomendaciones*.

**Keywords:** Age-Period-Cohort models, Bayesian inference, Bayesian MCMC, Fertility analysis

## Another proof of Pearson’s Theorem using symbolic computation in Python

Jose A Ortega, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Arturo Portnoy, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Wolfgang Rolke, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

In his groundbreaking 1900 paper, Karl Pearson introduced the chi-squared ( $\chi^2$ ) test, a fundamental tool that revolutionized statistics by providing a robust mathematical framework for assessing the goodness of fit between observed and expected frequencies. Over time, this tool has become foundational in data analysis and hypothesis testing across various disciplines, from social sciences to natural sciences.

While Pearson’s results were pioneering, various proofs and alternative derivations of his theorems have emerged. These approaches, including algebraic, geometric, and analytical methods, offer new perspectives and pedagogical insights into the core concepts introduced by Pearson.

With technological advancements, significant progress has been made in the tools available for tackling statistical problems. Programming languages, computational platforms, and

artificial intelligence have enabled mathematicians, scientists, and statisticians to extend the boundaries of symbolic and numerical computation. Languages like Python, along with specialized libraries such as SymPy, facilitate symbolic computation, enabling the discovery of patterns that were previously intractable. Once these patterns are identified, rigorous proofs can be established through inductive arguments or direct computations in general cases. Furthermore, platforms like Google Colab allow for intensive computations without the need to install software, using Google's servers to run the calculations in the cloud.

This talk presents a new proof of Pearson's theorem using Python, leveraging matrix properties and computational techniques. This modern approach provides an updated perspective on Pearson's results, adapted to contemporary technological tools, making their understanding and application more accessible in today's statistical analysis context.

**Acknowledgements:** BLA.

**Keywords:** Chi squared; Python, SymPy

## A probability model for *picas*, a Puerto Rican game of chance

Joseph Ficek, Oncology Statistics, GSK.

*Las picas de caballos* is a game of chance typically played at the *fiestas patronales* of Puerto Rico. It is a mechanical replica of a horse race, wherein numbered racing figurines rotate along circular tracks until randomly coming to rest. Players bet on the number of the figurine that stops closest to the finish line. Similar to roulette, the system of betting odds assumes that each figurine has an equal chance of winning. This is only the case, however, if the winner is determined by their angular distance from the finish line. If the winner is determined by circumferential distance (the distance along the length of the track), the probabilities of winning are no longer equal, and in fact depend on the relative sizes of the tracks. A probability model for the game is formalized. Implications for betting strategy and the house edge are discussed.

**Acknowledgements:** José A. Ortiz (Dameunbite Inc.) contributed to the conceptualization of this work.

**Keywords:** probability, gambling, games of chance, picas, fiestas patronales

# Converting Rule-Based Access Control Policies: From Complemented Conditions to Deny Rules

Josué A. Ruiz Rodríguez, Department of Mathematics, University of Puerto Rico at Humacao and Department of Computer Science, University at Albany SUNY.

Using access control policy rules with deny effects (i.e., negative authorization) can be preferred to using complemented conditions in the rules as they are often easier to comprehend in the context of large policies. However, the two constructs have different impacts on the expressiveness of a rule-based access control model. We investigate whether policies expressible using complemented conditions can be expressed using deny rules instead. The answer to this question is not always affirmative. In this paper, we propose a practical approach to address this problem for a given policy. In particular, we develop theoretical results that allow us to pose the problem as a set of queries to a SAT solver. Our experimental results using an off-the-shelf SAT solver demonstrate the feasibility of our approach and offer insights into its performance based on access control policies from multiple domains.

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**Keywords:** access control, complemented conditions, deny rules, negative authorization

## Simulación multinivel con ciclo de retroalimentación para optimizar un aparato fluido

José O. Sotero Esteva, Department of Mathematics, University of Puerto Rico at Humacao.

La optimización por ciclo de retroalimentación consiste de un proceso en el que el producto de un sistema es usado para retroalimentar el mismo sistema para optimizarlo según algún criterio. Ejemplos en la matemática son descenso por gradiente (ámpliamente usado en redes neuronales), método de Newton-Raphson y anilado simulado. Estos métodos aproximan de manera iterativa óptimos o raíces de alguna función  $f : D \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$ .

En el ámbito de la Ciencia de Materiales esta técnica toma la forma de experimentos autónomos que examinan de manera eficiente espacios de búsqueda multidimensionales con mínima intervención humana para encontrar formulaciones que optimizan propiedades de materiales. En este caso  $f = \Phi \circ \eta$  donde  $\eta : D \subseteq \mathbb{R}^n \rightarrow \mathcal{X}$  es un experimento hecho con parámetros físicos  $x_1, \dots, x_n$  y  $\Phi : \mathcal{X} \rightarrow \mathbb{R}$  es una *función de aptitud* que podría ser la medida tomada por un instrumento. Como las medidas y procedimientos experimentales tienen

ruido,  $\eta$  es una “función ruidosa”. Por tanto, los métodos frecuentemente seleccionados para optimizar  $f$  son de tipo Bayesiano.

Este trabajo estudia un método de separación de partículas llamado *fraccionamiento por flujo pinzado* (PFF). En este tipo de aparato partículas suspendidas en un líquido se introducen continuamente en un microcanal que tiene un segmento pinzado. Otro flujo de líquido sin partículas hace que las partículas inicialmente se alinean hacia una pared lateral en el segmento pinzado. Experimentalmente se ha corroborado que las partículas se desvían en direcciones distintas dependiendo de su tamaño al salir del canal pinzado. No está tan claro si la separación comienza dentro del canal.

Para este estudio se usa como experimento  $\eta$  una simulación de dinámica de partículas del canal estrecho del PFF. Los parámetros experimentales variados fueron el ancho  $a$  y longitud  $l$  del canal pinzado, las velocidades de entrada del solvente con partículas suspendidas  $v_m$  y del líquido sin partículas  $v_s$ , y parámetros  $\epsilon_m$  y  $\epsilon_s$  que determinan la cohesividad del solvente de las partículas y la del líquido sin partículas respectivamente. La función de aptitud  $\Phi$  se define basado en las posiciones finales de las partículas y sus tamaños. Dado que la porción relativa que ocupan inicialmente los dos líquidos dependen de  $v_m$  y  $v_s$ , se ha acoplado la simulación basada en partículas a una simulación inicial por método de elementos finitos para estimarlas.

Los resultados de este trabajo reflejan que la separación de partículas por tamaño comienza en el canal pinzado. Además se encontraron distintas combinaciones de valores  $a$ ,  $l$ ,  $v_m$ ,  $v_s$ ,  $\epsilon_m$ , y  $\epsilon_s$  que producen separaciones óptimas de las esferas por tamaño. Experimentos reportados en la literatura usan velocidades que satisfacen  $v_s/v_m = 10$ . Nuestro método encuentra valores con esa propiedad pero también señala otros tuplos con tasas de velocidades relativas mucho más bajas que podrán ser exploradas experimentalmente más adelante.

Este trabajo tiene dos méritos principales. El primero es el hallazgo de los nuevos parámetros experimentales. Por otro lado, aunque se puede encontrar en la literatura simulaciones de PFFs basadas en el método de elementos finitos, a nuestro conocimiento el uso tanto de simulaciones basadas en dinámica de partículas como la optimización por ciclo de retroalimentación son aplicaciones nuevas.

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**Palabras clave:** simulación multinivel, fraccionamiento por flujo pinzado, optimización bayesiana

## Autoencoder Teacher-Student model for flower tracking

Kenjiro García, Department of Computer Science, University of Puerto Rico at Río Piedras.  
Josué A. Rodríguez-Cordero, Department of Computer Science, University of Puerto Rico at Río Piedras.

Rémi Mégret, Department of Computer Science, University of Puerto Rico at Río Piedras.

Insects are the most important pollinator of crops and are essential for food security. Insect populations are declining in abundance and diversity and it's difficult to pinpoint the exact cause. Tracking pollinator visits may allow for more efficient crop production by assisting in their conservation efforts. Even though pollinator population monitoring is critically needed, traditional manual methods are very time consuming, invasive and/or disruptive. Our goal is to create an automated tool that can detect pollinator visits to flowers. As part of this system, it is required to detect and track the position of both insects and flowers. This work focuses on tracking all the flowers, which can correspond to hundreds of objects to track in each video frame. We propose an approach to reduce the computational burden by modeling the movement of all flowers based on the movement of a reduced number of them. The model has two-parts, based on an Autoencoder Teacher-Student neural network architecture. First an Autoencoder that transforms the positional information of all flowers of interest down into a lower-dimensional latent space that contains the relevant information needed to recreate the entire space of flower positions. Then a second model is trained to recreate this same latent space using only a small fraction of the original flower positions. Results showing the quality of the prediction will be shown and discussed.

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**Keywords:** Multi-object tracking, Autoencoder, Pollinator Monitoring

## Vision Transformer Models for Individual Re-Identification of Paint Marked Honeybees

Luke Meyers, Department of Computer Science, University of Puerto Rico at Río Piedras.

This work will present current results using vision transformer models for individual reidentification of honey bees in images. Previous work has demonstrated deep learning methods can re-identify both marked and unmarked bees from images, especially in the closed set: when evaluation only includes identities seen during training. However, with thousands of individuals in a single colony, models that can generalize to identify new individuals are sought.



Vision transformers are large deep learning models that can learn complex understandings of images, and have become the state of the art for tasks such as species classification. These models require a very large amount of data to learn, often mandating a general pre-training phase, and later fine-tuning process on data from the target domain.

Using a dataset of roughly 10,000 images across 64 individuals marked with unique paint codes, models were trained to learn a representative vector from each image that maps to a single identity. The effectiveness of these learned vectors for individual identification was tested on around 9000 images from 64 additional previously unseen individuals. Current best models achieved 83% accuracy given only one reference image per identity during evaluation. Comparisons in model pretraining and fine-tuning approaches will be presented, as well as visualizations of model error and identity understanding. Results will inform future research and applications of insect identification from images.

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**Keywords:** Computer Vision, Deep Learning, Re-Identification, Vision Transformers

## **El Centro Federal de Datos de Investigación Estadística en Puerto Rico (PR FSRDC)**

Mario Marazzi Santiago, Puerto Rico Federal Statistical Research Data Center, U.S. Census Bureau.

El PR FSRDC representa una apuesta que el Bureau federal ha hecho en el talento analítico de los investigadores de Puerto Rico. A través del Centro, los investigadores, ya sean estudiantes o profesores, pueden realizar investigaciones usando grandes bases de datos de una serie de entidades del gobierno de los Estados Unidos, incluyendo el Census Bureau, IRS, FRB, CDC, AHRQ, SAMHSA, BLS, BEA, entre otros. Ya contamos con una propuesta sometida por una profesora de la UPR, pero tenemos un tiempo limitado para demostrar el uso que los investigadores de Puerto Rico le darían a estas fuentes nuevas de información para investigaciones. El trabajo en un Centro de microdatos federal requiere conocimiento y destrezas analíticas sofisticadas, que incluyen la programación estadística usando varias herramientas disponibles en el Centro, tales como SAS, R, STATA, entre muchos otros.

# **Simulación de la Dinámica del Transporte Público Intermunicipal y su Impacto en la Toma de Decisiones de los Organismos de Tránsito: Caso de Estudio en el Departamento del Quindío**

*Jorge Mario García Usuga, Universidad del Quindío-Colombia.*

*Mónica Jhoana Mesa Mazo, Universidad del Quindío-Colombia.*

El transporte público intermunicipal es un componente esencial para la conectividad y el desarrollo regional en cualquier territorio. Sin embargo, su planificación y gestión enfrentan desafíos significativos, especialmente en la toma de decisiones informadas que optimicen el flujo vehicular y reduzcan los tiempos de viaje. Este estudio tiene como objetivo general simular la dinámica del transporte público intermunicipal y analizar cómo las decisiones de los organismos de tránsito impactan la eficiencia del sistema.

Para lograr ese objetivo, primero, se construyó una red compleja que represente la malla vial del transporte intermunicipal, modelando las rutas, nodos y conexiones entre distintas localidades. Después, se debe modelar la dinámica del transporte, considerando variables como la frecuencia de los viajes. Esto permitirá simular escenarios reales y predecir el comportamiento del sistema ante diferentes condiciones operativas.

Utilizando Python se integra la red compleja construida y la dinámica modelada con ecuaciones. Finalmente, se realizaron diferentes simulaciones computacionales las cuales permitieron conocer las medidas de centralidad de la red y evaluar los efectos de diferentes decisiones como la asignación de horarios de las empresas que prestan el servicio público de transporte y las frecuencias de viaje. Los resultados de estas simulaciones se contrastaron con datos de tiempos de viaje obtenidos de aplicaciones de navegación en tiempo real y sistemas GPS. Como caso de estudio, esta metodología fue aplicada en el departamento del Quindío, Colombia, con el fin de evaluar su efectividad en un contexto regional específico y explorar su potencial de implementación en otras regiones.

## **Análisis estadístico de los factores sociales que influyen en el rendimiento académico de los estudiantes de la FAE, UPRRP**

*Oscar Y. Castrillon-Velandia, Instituto de Estadística y Sistemas Computarizados de Información, Facultad de Administración de Empresas, Universidad de Puerto Rico en Río Piedras.*

En esta charla discutiremos la influencia de diversos factores sociales en el rendimiento académico de los estudiantes de nuevo ingreso en la Facultad de Administración de Empresas de la Universidad de Puerto Rico (FAE), Recinto de Río Piedras. Para ello, se empleó un enfoque estadístico descriptivo e inferencial, analizando datos de estudiantes matriculados entre los años académicos 2018-2019 y 2022-2023. Se examinaron variables como el tipo de escuela de procedencia (pública o privada), el estatus de dependencia económica, la ocupación

de los padres y la región educativa de origen.

Los resultados indican que los estudiantes con dependencia económica obtuvieron un mejor desempeño académico en comparación con aquellos económicamente independientes. Asimismo, se observó una disminución en las habilidades matemáticas de los nuevos estudiantes, lo que ha afectado su rendimiento en cursos clave como Métodos Cuantitativos para Administración de Empresas. El estudio también destaca un descenso consistente en el rendimiento académico en los últimos seis años.

El estudio subraya la necesidad de intervenciones específicas para abordar las disparidades sociales y mejorar el éxito de los estudiantes.

**Agradecimientos:** Este trabajo fue realizado en colaboración con Daiver Velez-Ramos (Universidad de Puerto Rico en Río Piedras), Jairo A. Ayala-Godoy (Universidad de Puerto Rico en Río Piedras), Rafael Aparicio-Cuello (Universidad de Puerto Rico en Ponce), y Edwin Flórez (Universidad de Puerto Rico en Mayagüez). Esta investigación fue financiada por el Research Award Program FAE 2022-2023 de la Facultad de Administración de Empresas, y por los Fondos Institucionales para la Investigación (FIPI) 2023-2025, otorgados por el Decanato de Estudios Graduados e Investigación (DEGI).

**Palabras clave:** factores sociales, rendimiento académico, análisis descriptivo, análisis inferencial

## Is the Puerto Rico coastline a rectifiable curve or a factorial or both or neither?

*Peter Cholak*, Department of Mathematics, University of Notre Dame, Notre Dame, IN.

What is the Jones  $\beta$  number (at scale  $r$ ) of the coastline? What Hausdorff dimension of the coastline? Does any of this help us understand or control coastline erosion? While we will not exactly answer these questions we will explain what all these terms mean. Hopefully we will encourage everyone to learn more geometric measure theory! This will be a purely expository talk. But the speaker is always willing to talk (offline) about his related research.

## $L^p$ characterization of well posedness for second-order abstract differential equations

*Rafael Aparicio*, Department of Mathematics, University of Puerto Rico at Ponce.

*Valentin Keyantuo*, Department of Mathematics, University of Puerto Rico at Río Piedras.

Given a closed and densely defined linear operator  $A$  on a Banach space  $X$ , we give characterizations for the well-posedness of the Dirichlet boundary value problem for the homogeneous

linear second order differential equation:  $u''(t) + Au(t) = 0$ , is  $L^p$ -well-posed for  $1 \leq p < \infty$ .

**Keywords:** Second order differential equations; Dirichlet boundary value problem;  $L^p$ -well-posedness

## 4 Afiches / Posters

(In alphabetical order using the first name of the presenter.)

### **A low rank Gaussian process prediction model for very large datasets**

Alexis Javier Aguirre Narváez, Universidad de Puerto Rico en Mayagüez.

The Gaussian process prediction model requires expensive computation to invert the covariance matrix it depends on and also has considerable storage needs. A recent method for very large spatial data known as Fixed Rank Kriging allows for prediction when the Gaussian process prediction model cannot and is easily implemented with less assumptions about the process. However, Fixed Rank Kriging requires the estimation of a matrix which must be positive definite, and the original estimation procedure cannot guarantee this property. We present a result that shows when a matrix subtraction of a given form will give a positive definite matrix. Motivated by this result, we present an iterative Fixed Rank Kriging algorithm that ensures positive definiteness of the matrix required for prediction and show that under mild conditions the algorithm numerically converges. The new Fixed Rank Kriging procedure is implemented to predict missing chlorophyll observations for very large regions of ocean color. Predictions are compared to those made by other well known methods of spatial prediction.

### **Using Beehive Video Monitoring Data for Individual Bee Travel Construction and Analysis**

Andrea V. Nieves Rivera, Department of Computer Science, University of Puerto Rico at Río Piedras.

Marie Lluberes Contreras, Department of Computer Science, University of Puerto Rico at Río Piedras.

Bees are an essential part of human agriculture due to being the primary pollinators for about a third of necessary crops. They are also the easiest pollinators to introduce to an agriculture site, leading them to being popular among cultivators. However, much about their behavior and patterns is yet unknown. For the purpose of discovering and analyzing

new information, scientists have taken measures to record the bees' activities at the entrance to the colony. Afterwards, data is extracted from these videos with a general idea of the location and trajectory of each bee's appearance on video. The data, however, lacks proper classification and processing to meaningfully express the flight tendencies at the entrance of the hive. Furthermore, the amount of data extracted from the videos is so large that manually classifying every entry is unfeasible. The purpose of this project is to create an algorithm to automatically clean and process the raw data extracted from these videos at a large scale to visualize a timeline of a bee's activities is more feasible as well as providing a statistical summary of an individual bee's tendencies. An algorithm is used to assign location estimates to many data entries where the location is unclear. Subsequently, another algorithm is utilized to classify different entries as an action of entering or leaving the hive, based on the location estimates of subsequent data entries. Finally, a chronogram describing the general location of the bee is made with the locations as a predictor for trajectory, giving a clear notion on a bee's entrances and exits. This strives to serve as a tool for biologists to find tendencies and patterns, as well as discover new concepts about bees based on their flight times and durations.

**Acknowledgements:** This material is in part based upon work supported by the National Science Foundation under Grant No. 2318597.

**Keywords:** video data analysis, data visualization, bee behavior

## **Acyclic subgraphs and its role on the Transient of Boolean Monomial Dynamical Systems**

Bryan C. Busby Polanco, Electrical Engineering Department, University of Puerto Rico at Mayagüez.

*Omar Colón-Reyes*, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

An important problem in the theory of finite dynamical systems is understanding how the structure of a system determines its dynamics. In this paper, we study acyclic subgraphs contained in dependency graphs induced by boolean monomial systems. Results by O. Colón, et. al, relates the number of state transitions to walks in the dependency graph of the system. We are interested in the role that plays acyclic subgraphs, in the dependency graph, over the behavior of the system. In particular, we have observed that for certain acyclic graphs, we can obtain a family of linear equations, whose solution appears to provide an insight of the transient of the system itself.

# A Metadata-Driven Framework for Managing High-Throughput Sensor Data in Ecological Edge Networks

*Carlos J. Perez-Vinelli*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*Carlos J. Corrada Bravo*, Department of Computer Science, University of Puerto Rico at Río Piedras.

*Remi Megret-Laboye*, Department of Computer Science, University of Puerto Rico at Río Piedras.

Ecological monitoring is vital for understanding and managing natural ecosystems, yet rapid advances in sensor technology have led to higher data volumes that challenge traditional data management approaches. High throughput sensor networks, used for collecting environmental data, can now generate data streams over 5 TB per week, placing heavy demands on centralized, cloud dependent systems that struggle with high latency, limited bandwidth, and increased energy consumption. Organizing and storing data manually can be a time-consuming task, prone to errors, and can lead to inconsistent data standards. Although edge computing has reduced some issues by processing data locally, these implementations still see scalability issues and metadata management challenges in heterogeneous sensor networks.

To address these challenges, we propose the integration of a policy and metadata driven data management platform into our existing data pipeline. Our approach integrates automated data annotation and the enforcement of data lifecycle policies.

By implementing this framework, we expect large volumes of data to be structured through automated data annotation for ease of querying and retrieval. Lifecycle policies being enforced will make sure data integrity is safeguarded against corruption or data loss. Additionally, the automation of routine data management tasks is expected to at least reduce manual intervention, permitting researchers to focus on higher level tasks.

This policy and metadata driven platform will improve scalability, make ecological data workflows more efficient, and make data management reliable, facilitating a more effective ecosystem monitoring and scientific discovery.

**Keywords:** Ecological Monitoring, Sensor Networks, Edge Computing, Metadata Management, High-Throughput Data Processing

## A control theory for monomial dynamical systems over Finite Fields

Dennis E. Quintano Villanueva, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Omar Colón-Reyes, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Criteria to determine when a Monomial Dynamical System over a finite field is a fixed point system have been determined by O. Colón et. al. Such results could be use to determine when a monomial dynamical control system can be stabilizable. A monomial control system is a function  $F : \mathbb{Z}_p^n \times \mathbb{Z}_p^m \rightarrow \mathbb{Z}_p^n$ , such system is stabilizable if there exists a map  $g : \mathbb{Z}_p^n \rightarrow \mathbb{Z}_p^n \times \mathbb{Z}_p^m$  such that  $F \circ g$  is a fixed point system. In this work we will present a family of control systems that can be stabilizable and present necessary conditions for a system, in general, to be able to stabilize.

## The DHARMa Library for the Analysis of Residuals in Generalized Linear Models in R

Henry A. Molina, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Raúl E. Macchiavelli, College of Agriculture Sciences, University of Puerto Rico at Mayagüez.

We discuss the analysis of residuals in generalized linear models (GLM) and how they help to validate a model determining in a data set or experiment, leading to more solid inferences. We find that limitations such as overdispersion, inflation of zeros, or misspecification of distributions can be diagnosed using DHARMa. Simulated examples are used to explore the different advantages in the GLM by implementing the DHARMa library and explore situations in which it is more helpful to diagnose assumptions.

indent **Acknowledgements:** BLA.

**Keywords:** DHARMa, GLM, Residual analysis

## Coale-McNeil Nupciality Model: Age at First Marriage in Puerto Rico

Jan L. Carrasquillo López, Department of Mathematics, University of Puerto Rico at Río Piedras.

This study uses data from the Puerto Rico Reproductive Health Survey, 1995-96 to identify patterns in the age distribution at first marriage in Puerto Rico using the Coale-McNeil

nuptiality model. The model allows us to describe and predict the age at first marriage by analyzing data on age at first marriage. Specifically, we explore how the distribution of first marriages can be represented using three key parameters: the proportion of women who eventually marry, the mean and standard deviation of the age at first marriage. The Coale-McNeil model employs an inverse gamma function, with parameters controlling the location, scale, and proportion of the cohort that marries. Our analysis describes nuptiality levels and trends during the 1980's and 1990's in Puerto Rico. For our results, the model initially did not fit the data well when considering the complete dataset, suggesting that the chosen parameters may not accurately represent the marriage distribution in Puerto Rico. However, when working with smaller datasets, such as regional analysis, the model performs well. This indicates that while the model may be effective for smaller subsets of data, it may require adjustments to improve accuracy when applied to the larger population.

**Acknowledgements:** This is joint work with Luis R. Pericchi Guerra (University of Puerto Rico at Río Piedras), Angélica M. Rosario Santos (University of Puerto Rico at Río Piedras), and Hernando Mattei Torres (University of Puerto Rico Medical Sciences Campus). I would like to express my deepest gratitude to my mentors Dr. Pericchi, Dr. Mattei and Dr. Rosario, and the University of Puerto Rico, for their invaluable support and guidance throughout this research.

**Keywords:** Coale-McNeil nuptiality model, age at first marriage, Puerto Rico, fertility data, demographic analysis

## **Extremal Spectral Radii of Arithmetical Structures on Bident and Star Graphs**

*Jean García-Colón*, University of Puerto Rico at Mayagüez.

An arithmetical structure is a labeling of integers on a finite, connected graph such that each vertex divides the sum of its neighbors, and the greatest common divisor among all labels is 1. There is a matrix associated to each structure that encodes these divisibility conditions. The goal of this project was to find the structures that maximize and minimize the spectral radii of arithmetical structures on bident and star graphs. In this poster, we present the structure that minimizes the spectral radius on bident graphs and a conjecture about which structures maximize the spectral radius on bident graphs. For star graphs, we show which structures minimize the spectral radius when the number of vertices is a perfect square or a pronic number, and we present a conjecture for which structures minimize all others.

**Acknowledgements:** This is joint work with Katie Noonan, Katie Shattuck, and Alexander Diaz-Lopez. We thank our advisor Dr. Diaz-Lopez for his help and guidance during this project, Dr. Joel Louwsma for his useful comments, Villanova's Co-MaStER program for providing the opportunity to engage in research at Villanova University, Augie team for



providing us access to a supercomputer, and NSF-DMS 2211379 for providing support.

**Keywords:** arithmetical structures, spectral radii, bident graphs, star graphs

## **Comparando la biodiversidad de tres áreas con diferentes niveles de estado de restauración en la Reserva Natural Ciénaga Las Cucharillas utilizando los índices de Shannon-Weaver, Margalef y Simpson**

*Nicole Merced*, Departamento de Ciencias, Escuela con Causa Rosalina Caraballo.

*Juan J. Nieves Álvarez*, Coordinador de Programa de Ecología, Caras con Causa.

Los humedales son ecosistemas vitales que albergan una increíble diversidad de vida, desempeñando un papel fundamental en la salud de nuestro planeta. El estudio de la biodiversidad en estos entornos acuáticos no solo nos permite comprender la complejidad de las interacciones entre especies, sino que también nos brinda información invaluable para la conservación y el manejo sostenible de estos ecosistemas. En este trabajo se busca explicar la importancia de preservar y estudiar la biodiversidad en los humedales, destacando su relevancia para la salud ambiental. La identificación de bioindicadores ecológicos en los monitoreos de biodiversidad es fundamental para comprender la salud de los ecosistemas, las interacciones biológicas y la planificación de estrategias para dirigir esfuerzos de conservación.

Se realizaron censos de biodiversidad de fauna en tres áreas con diferentes niveles de estado de restauración en la Reserva Natural Ciénaga Las Cucharillas en Cataño los cuales representaban esfuerzos luego de 10 años (siembra de Vereda Principal), 4 años (siembra Motorambar) y 1 año (siembra Planeta Tierra). Los índices de biodiversidad realizados mostraron baja y moderada diversidad de especies en las respectivas áreas de estudio (Shannon-Weaver: 1.25, 1.28, 1.59; Margalef: 1.41, 1.48, 1.49; Simpson: 0.54, 0.67, 0.76). Los datos muestran que la reinita común, la mariposa sulfur y el lagartijo común están presentes en las tres áreas evaluadas. Entendemos que según las características de las áreas de estudio, la mariposa sulfur sirve como indicador del estado de restauración de estas áreas reforestadas y de igual forma entendemos que la reinita común aparenta interactuar con mayor frecuencia en áreas con árboles maduros los cuales producen mayor cantidad de alimento a estas.

**Agradecimiento:** Este estudio fue realizado por estudiantes de escuela intermedia de la Escuela con Causa Rosalina Caraballo y patrocinado por el Laboratorio Comunitario de Empoderamiento Científico y Cultural (LabCom) de Caras con Causa.

**Palabras clave:** conservación, humedales, índices de biodiversidad

## **Análisis de las curvas de descenso más rápido en la topografía puer- torriqueña utilizando datos de elevación**

*Keliel J. Soto Ortiz*, Departamento de Matemáticas, Universidad de Puerto Rico en Humacao.

*Pablo V. Negrón Marrero*, Departamento de Matemáticas, Universidad de Puerto Rico en Humacao.

El problema de la braquistócrona sobre una superficie consiste en buscar o calcular la curva sobre dicha superficie por la cual una partícula se desliza de un punto de la superficie a otro, sin fricción y bajo los efectos de la gravedad, en tiempo mínimo. Desde el punto de vista matemático esto se presenta como un problema del cálculo de variaciones, donde minimizamos el funcional o integral de tiempo. En particular nos interesó modelar la superficie del Bosque Nacional el Yunque en Puerto Rico utilizando datos discretos de elevación (tipo DEM) obtenidos del Servicio Geológico de los Estados Unidos (USGS). A partir de estos datos DEM, obtuvimos una versión discreta de la función (de dos variables) que describe la superficie del Yunque. Esta función discreta la pudimos evaluar y también aproximarle sus derivadas parciales, utilizando la función especializada “griddata” del programado MATLAB. Luego de discretizar la integral de tiempo, implementamos una generalización del método de descenso máximo para aproximar el mínimo de la integral de tiempo, el cual corresponde a una aproximación de la braquistócrona buscada. Estaremos presentando los resultados obtenidos de este trabajo para uno de los picos del Yunque. Esta investigación nos abre un campo amplio de cómo el análisis topográfico puede combinarse con herramientas de programación avanzadas para resolver problemas de relieve en superficies reales como las de Puerto Rico.

**Palabras clave:** braquistócrona, datos DEM y UTM, interpolación

## **Localización de Colmenas de Abejas Silvestres: Un Enfoque Estadístico para Comprender el Comportamiento de las Abejas**

*Lizbeth Alvarado-Vargas*, Facultad de Educación, Universidad de Puerto Rico en Río Piedras.

La localización de panales de abejas silvestres es fundamental para la investigación ecológica y agrícola, ya que permite analizar el comportamiento de las abejas y su contribución a la polinización. Este estudio tiene como objetivo estimar la ubicación de los panales de abejas silvestres mediante un proceso de etiquetado de abejas, análisis de registros de tiempos de retorno y direcciones de vuelo. Con estos datos recolectados en diversos puntos de la isla, se aplicarán técnicas de triangulación apoyada de análisis estadísticos avanzados y modelización de patrones de movimiento para aproximar con precisión la localización de las colmenas.

Además, se registran variables ambientales (temperatura, humedad relativa e intensidad

lumínica), con las cuales se analizara el comportamiento de las abejas mediante análisis de correlación y modelos de regresión. Estos análisis facilitan la comparación entre individuos y colmenas, así como la identificación de tendencias relevantes en los datos.

Este proyecto se implementará en diversas escuelas de Puerto Rico, donde los estudiantes recolectarán y analizarán datos, promoviendo la alfabetización científica, el pensamiento crítico y la aplicación práctica de métodos estadísticos en la resolución de problemas ambientales.

**Agradecimientos:** Este trabajo fue realizado en colaboración con Ariana I. Rodríguez-Flores, Martin Geria-Reines, Josué Rodríguez, Luis F. Aparicio-Mestra, Ernesto Espada-Nazrio, Amilcar Vélez-Flores, Jaime W. Abreu-Ramos, Remi Megret, Tugrul Giray, José L. Agosto-Rivera, José M. Álvarez, y Jairo A. Ayala Godoy, todos de la Universidad de Puerto Rico en Río Piedras. Agradecemos a la National Science Foundation (NSF) por su apoyo financiero a través de las subvenciones:(2321760, 2216584, 2215753, 2318597).

**Palabras Clave:** Colmenas de abejas silvestres, localización, geolocalización, regresión y análisis de correlación

## Uso del número de Frobenius para determinar el *transient* de sistemas dinámicos de punto fijo sobre campos finitos de característica 2

Mario Jacobo Motiño Palma, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Omar Colón-Reyes, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Un sistema dinámico monomial booleano es una función  $f = (f_1, \dots, f_n) : \mathbb{Z}_2^n \rightarrow \mathbb{Z}_2^n$ , donde  $f_i = x_1^{\epsilon_{i1}} x_2^{\epsilon_{i2}} \cdots x_n^{\epsilon_{in}}$  no es constante y al menos un  $f_j$  depende de dos o mas variables. Mas aún, asociamos a  $f$  un grafo de dependencia, cuyo conjunto de vértices es  $\{a_1, a_2, \dots, a_n\}$  y  $a_i \rightarrow a_j$  si  $x_j | f_i$ . Queremos hallar el mínimo entero  $t > 0$ , llamado *transient*, tal que si  $f$  es un sistema de punto fijo,  $f^t = f$ . Esto es, queremos saber cuanto tarda  $f$  en estabilizarse. Nuestro objetivo es vincular la estructura de  $f$  con el transient, esto es, como podemos saber el transient a partir de la estructura de  $f$ , en particular el grafo de dependencia, sin tener que enumerar todos los posibles estados.

Este trabajo provee una respuesta para una familia de sistemas cuyo grafo de dependencia es el “wedge” de tres ciclos de longitud 3,  $m$  y  $n$ , respectivamente, donde  $m$  y  $n$  son primos,  $n < m$ .

Sorpresivamente, descubrimos que para hallar el transient de  $f$  debemos computar, para algunos casos, el numero de Frobenius para tres números enteros. En el 2017, Tripathi (Journal of Number Theory 170 (2017)), provee fórmulas cerradas para el número de Frobenius de tres enteros. Usando estos resultados, podemos proveer explícitamente una fórmula para

el transient.

## A Bayesian Approach to modeling Fertility Predictors in Puerto Rico using Logistic Regression

Nathalie N. González Narváez, Department of Mathematics, University of Puerto Rico at Río Piedras.

Logistic regression models are widely used in demographic studies. We propose to apply a Bayesian approach to model selection under logistic regression to explore fertility patterns using demographic indicators from the *Puerto Rico Community Survey* and *American Community Survey* from 2005-2022. We define our research question as a binary response: whether or not had a child in the last twelve months, based on predictors such as marital status, educational level, age, poverty level, labor participation, year, and place of residence (Puerto Rico vs. U.S. states).

For model selection, we implement Bayesian strategies such as Posterior Inclusion Probabilities (PIP), the Median Probability Model (MPM), and the Highest Posterior Probability Model (HPM). We also briefly explore Bayesian model averaging to improve predictive performance. This probabilistic framework offers a robust alternative to classical methods, providing deeper insights into fertility trends in Puerto Rico. Preliminary findings indicate that age, marital status, poverty level, educational level, and year are the most influential predictors of fertility outcomes. In contrast, labor participation does not significantly affect fertility in the best-fitting model. Additionally, this methodology provides a probabilistic foundation for forecasting future fertility trends under different scenarios. Future research will expand on these findings by examining regional differences in fertility patterns. All the required programming was done in **R** software and **BAS** package was used for fitting models and calculating their posterior probabilities. This approach enables us to quantify uncertainty, incorporate prior knowledge, and enhance predictive accuracy, particularly when working with limited data.

**Acknowledgements:** This is joint work with Angélica M. Rosario Santos (University of Puerto Rico, Medical Sciences Campus), Luis R. Pericchi Guerra (University of Puerto Rico at Río Piedras), María Pérez Hernández (University of Puerto Rico at Río Piedras), and Hernando Mattei (University of Puerto Rico, Medical Sciences Campus). This research is sponsored by *Estudio Demográfico de Puerto Rico 2024: Análisis y Recomendaciones*.

**Keywords:** Logistic Regression models, Bayesian inference, Demographic inference, Fertility determinants

# Modeling and Statistical Analysis of Bacterial Growth Dynamics in Honey Bee Colonies: An Interdisciplinary Approach

Norangelik Quiles Soto, Department of Biology, University of Puerto Rico at Río Piedras.

Our laboratory has identified complex interactions between bacterial communities present in the brood caps of honey bee colonies and the maturation process of brain cells within specific neural circuits. This discovery, which suggests that brood caps may serve as an important source of beneficial probiotics, has motivated the creation of a bacterial library extracted from natural populations. The characterization of this library is being carried out on multiple levels: genetic, metabolic, morphological, and physiological.

From the perspective of mathematics and statistics, our project incorporates the development and validation of bacterial growth models to fit and predict proliferation dynamics under controlled conditions. To this end, we have implemented an optimized high-throughput method using a plate reader (Tecan Infinite 200 Pro) that automates the collection of optical density (OD at 600 nm) measurements at regular intervals (four measurements per hour). The optimal experimental parameters (35 °C, orbital shaking with a 3 mm amplitude, a well volume of 200  $\mu$ l in 96-well plates) were determined based on preliminary analyses and adjustments grounded in experimental design and statistical methods.

The modeling phase focuses on evaluating different growth curves, including logistic, Gompertz, and other parametric and non-parametric approaches. Statistical inference techniques are applied to estimate model parameters, assess goodness-of-fit (using criteria such as AIC, BIC, and residual analysis), and validate the robustness of predictions via resampling and cross-validation methods. In addition, numerical optimization methods are explored to determine optimal growth times that ensure the desired production of probiotics, incorporating sensitivity analyses and stochastic simulations to address the inherent uncertainty in the experimental data.

The integration of mathematical models and statistical tools not only optimizes experimental design and data interpretation but also paves the way for applying these approaches to assess the influence of environmental factors (pesticides, antibiotics, and contaminants) on the behavioral development of honey bees. Ultimately, this interdisciplinary approach has the potential to contribute significantly to mitigating global honey bee colony losses a crucial challenge for food security.

**Acknowledgements:** This is joint work with Cristina L. Andujar Sierra, Miguel A. Urdaneta Colón, Yilmaz Berk Koru, Tugrul Giray, Abigail Strubbe Nieves, Jairo A. Ayala-Godoy, and José L. Agosto-Rivera, all from the University of Puerto Rico at Río Piedras. This research is sponsored by the National Science Foundation (2216584, 2321760, 2215753, 2318597).

**Keywords:** Honey Bee Colonies, Bacterial Growth Dynamics, Mathematical Modeling, Statistical Analysis, Probiotics, Growth Curve Modeling

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## Contact Us:

- Biostatistics MS Program, Director  
William Fadel, wffadel@iu.edu
- Biostatistics MS Program, Coordinator  
Ziyi Yang, ziyiyang@iu.edu
- Website: <https://go.iu.edu/8qJ4>



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