Contents

1 Charlas Plenarias / Plenary Talks

Fractal zeta functions and fractal drums: an introduction to the theory of complex dimensions

Michel L. Lapidus, Department of Mathematics, University of California, Riverside.

We will give some sample results from the new higher-dimensional theory of complex fractal dimensions developed jointly with Goran Radunovic and Darko Zubrinic in the recently published nearly 700-page research monograph (joint with these same co-authors), Fractal Zeta Functions and Fractal Drums: Higher Dimensional Theory of Complex Dimensions [2], published by Springer in February 2017 in the Springer Monographs in Mathematics series. We will also explain its connections with the earlier one-dimensional theory of complex dimensions developed, in particular, in the research monograph (by the speaker and M. van Frankenhuijsen) entitled "Fractal Geometry, Complex Dimensions and Zeta Functions: Geometry and Spectra of Fractal Strings" [1] (Springer Monographs in Mathematics, Springer, New York, 2013; 2nd rev. and enl. edn. of the 2006 edn.).

In particular, to an arbitrary compact subset A of the N-dimensional Euclidean space (or, more generally, to any relative fractal drum), we will associate new distance and tube zeta functions, as well as discuss their basic properties, including their holomorphic and meromorphic extensions, and the nature and distribution of their poles (or 'complex dimensions'). We will also show that the abscissa of convergence of each of these fractal zeta functions coincides with the upper box (or Minkowski) dimension of the underlying compact set A, and that the associated residues are intimately related to the (possibly suitably averaged) Minkowski content of A. Example of classical fractals and their complex dimensions will be provided.

Finally, if time permits, we will discuss and extend to any dimension the general definition of fractality proposed by the author (and M-vF) in their earlier work [1], as the presence of nonreal complex dimensions. We will also provide examples of hyperfractal, for which the critical line Re(s)=D, where D is the Minkowski dimension, is not only a natural boundary for the associated fractal zeta functions, but also consist entirely of singularities of those zeta functions.

Fractal tube formulas are obtained which enable us to express the intrinsic oscillations of fractal objects in terms of the underlying complex dimensions and the residues of the associated fractal zeta functions. Intuitively, the real parts of the complex dimensions correspond to the amplitudes of the associated geometric waves, while their imaginary parts correspond to the frequencies of those waves. This is analogous to Riemann explicit formula in analytic number theory, expressing the counting function of the primes in terms of the underlying zeros of the celebrated Riemann zeta function. These results are used, in particular, to show the sharpness of an estimate obtained for the abscissa of meromorphic convergence of the spectral zeta functions of fractal drums. Furthermore, we will also briefly discuss recent joint results in which we obtain general fractal tube formulas in this context (that is, for compact subsets of Euclidean space or for relative fractal drums), expressed in terms of the underlying complex dimensions. We may close with a brief discussion of a few of the many open problems stated at the end of the aforementioned book.

Exploring, generalizing and applying the covering method

Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

The divisibility of exponential sums has been used to characterize and prove properties in coding theory, cryptography and solvability of polynomial equations. In general, algebraic methods to estimate the p-divisibility of exponential sums over finite fields are non-elementary. The covering method provides an elementary and intuitive way to determine p-divisibility, which is particularly convenient in the applications. In this talk we will give an overview of the covering method for computing the p-divisibility of exponential sums and explain how it can be used in some applications.

Mathematical Models and Emerging Infectious Diseases

Abdul–Aziz Yakubu, Department of Mathematics, Howard University.

The number of outbreaks of emerging infectious diseases is growing and zoonotic diseases, which can be spread between animals and humans, continue to represent well over half of all infections in humans. In this talk, we will highlight some of the 25 deadliest diseases in human history and the top 10 most deadly diseases in Africa. In case studies, we will introduce mathematical models of Ebola and malaria, and illustrate how they might be used to mitigate outbreaks of these diseases and other pathogens.

2 Panel de Discusión / Discussion Panel

2.1 El Componente de Matemática en los Currículos Universitarios: Cálculo vs Estadística vs Otros

Jorge M. López, Universidad de Puerto Rico en Río Piedras.
Orville Disdier, Instituto de Estadísticas de Puerto Rico.

Tradicionalmente, el estudio del cálculo ha sido favorecido como el componente de matemáticas de educación general. En la actualidad, la mayoría de las disciplinas requieren de las herramientas de las estadísticas para el avance de las mismas. Sin embargo, muy pocos programas las contemplan en sus currículos. El panel pretende traer a discusión de los panelistas y la audiencia el contenido del componente de matemáticas en los currículos universitarios.

3 Charlas Concurrentes / Concurrent Talks

(In alphabetical order using the last name of the speaker.)

Numerical simulations for turbulent drag reduction using liquid infuse surface

Isnardo Arenas–Navarro, The University of Texas at Dallas.
Stefano Leonardi, The University of Texas at Dallas.

Turbulent drag reduction has been the goal of many research studies with the aim of minimizing fuel consumption and pollution, and increasing manoeuvrability in vehicles. To obtain drag reduction active and passive approaches have been used.

In the active approach, a control strategy is used to change the generating and sustaining turbulence mechanism. Usually this requires energy to move actuators which are comparable to the drag reduction obtained. Passive control instead does not require an external activation and consists generally on modifying the surface geometry. Recently, Liquid Infused Surfaces are getting relevance as a passive approach. They consist of a film of lubricating oil locked in place by a micro/nanoporous substrate. The drag reduction is obtained thanks to a slip velocity at the interface that reduces the shear in the surface.

Liquid infused surfaces are typically modelled as regions of alternating non-slip and free-slip boundary conditions. This is not realistic because the shear is reduced but is not zero, as the free-slip assumption implies. In addition, the porous media induces a form drag and the interface between the fluids is deformed rather than flat as usually modelled. We overcame these limits by developing a numerical code which combines an immersed boundary method to mimic the substrate and a level set to track the interface. We performed direct numerical simulations of two superposed fluids in a turbulent channel with a texture surface consisting in longitudinal square bars and staggered cubes. The viscosity of the fluid inside substrate is smaller than that of the main stream ($m = \frac{\mu_1}{\mu_2} = 0.1, 0.4$ where the subscripts 1 indicate the fluid in the substrate and 2 indicate the fluid in the main stream). Two case are compared varying the Weber number: $We = 0$, implying the interface remains flat due the surface tension, and $We = 1000$, in this case the interface can be displaced due the normal velocity to the interface. We will discuss at the meeting how the drag is reduced by the Liquid Infused surfaces and how it depends on the geometry of the cavities and on the surface tension.

Acknowledgements: Numerical simulations were performed on XSEDE TACC under Grant CTS070066. This work was supported by ONR MURI grants N00014-12-01-0875 and N000014-12-01-0962.

Keywords: numerical methods, computational fluid dynamics, level set method, turbulent flows, immersed boundary method, drag reduction.

Recent developments in the homotopy groups of spheres

Mark Behrens, Department of Mathematics, University of Notre Dame, Notre Dame, IN, USA.

I will discuss some recent developments in the computation of the stable homotopy groups of spheres ($\pi_{n+k}(S^k)$ for k large). These computations have implications to the existence of exotic spheres in dimension 61 and beyond.

Acknowledgements: This research was partially supported by the National Science Foundation.

Keywords: homotopy groups, homology, smooth structures.

Combinatorial Problems in the Mathematical Olympiad of Central America and the Caribbean

Luis F. Cáceres–Duque, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Rafael Sánchez–Lamoneda, Universidad Antonio Nariño, Bogotá, Colombia.
José H. Nieto–Said, Universidad del Zulia, Maracaibo, Venezuela.

In this talk we analyze the combinatorial problems proposed of the Mathematical Olympiad of Central America and the Caribbean, during its eighteen years of existence. The different types of combinatorial problems (counting, existence, strategy games, etc.) are explained and illustrated with various examples. Some original problems submitted to the Olympiad but not selected in the papers, are also discussed.

Keywords: Olympiad, combinatorics, problem, solution.

The peace movement paradox

Dennis G. Collins, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

This talk follows up a presentation "A conjectured hypercube invariant in generalized gravity" Oct. 8, 2016 at the Indiana MAA (Math Assoc. of America) Section meeting at Purdue University, West Lafayette, IN, which derived a gravitational potential for how maximum entropy solutions can fluctuate around an equilibrium with points being successively pulled closer together (peace) or drifting apart (war). Peace movements put more energy into the potential well that determines oscillations or pulsing around maximum entropy (=minimum Fisher information) point(s) at the bottom of the well. This added energy means greater excursions in the direction of more unity (peace) but ALSO (with pulsing) greater excursions away from unity toward greater conflict. As examples the Stockholm Olympics of 1912 were followed in 1914 by World War I, the Berlin Olympics of 1936 were followed by World War II in 1939 and the Sochi Olympics of 2014 were/are followed by the Crimea and Syrian conflicts. At present there is a similar danger of the Rio de Janeiro Olympics

and Columbian peace deals being followed by increased conflict. R.J. Rummel (The Conflict Helix) tried to deal with this situation in terms of war and peace being equilibrium points and catastrophe theory explaining transitions. Since the harmonic oscillator spends most time (probability) at the turning points, there is some reason for this point of view; however the theory has not produced a convincing helix whereas the author's theory does, including the so-called "shearing" of the helix. Mathematical models involve adding so-called "damping" or "ramping up" terms. There is also the increased threat of nuclear war with the 1962 Cuban missile crisis following the 1960 Rome Olympics and the Pope's "Pacem in Terris" communication.

Las matrices en la adquisición de datos compresiva

Edwin Flórez, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

En muchos problemas de la vida real nos encontramos que queremos inferir ciertos valores que nos interesan haciendo muestreo. Un caso típico es en el procesamiento de señales en donde se busca reconstruir una señal teniendo en cuenta los datos observados. Si el proceso de adquisición de los datos es lineal, el problema se reduce en solucionar un sistema de ecuaciones lineales. Matemáticamente los datos observados y_m estarán asociados con la señal que nos interesa x_N por medio de $Ax_N = y_m$. El proceso de recuperar la señal reduce a solucionar este sistema y como sabemos, si $m < N$, el sistema es indeterminado y por consiguiente tiene infinitas soluciones. Para la recuperación se necesitará información adicional. Aquí es donde precisamente ayuda el Teorema de Nyquis–Shanom, el cual dice que la tasa de muestreo de una señal continua en el tiempo debe ser dos veces su frecuencia mas alta para asegurar la reconstrucción.

Cuando en 2005, Emmanuel Candès, Terence Tao y David Donoho publicaron que bajo ciertas asunciones era posible reconstruir la señal con valores de m muy por debajo de N y sin contradecir el Teorema de Nyquis–Shanom, fue una tremenda sorpresa y empezó una revolución en la comunidad científica para su evolución e inclusión en diversos campos. Esta técnica reduce el tiempo consumido al analizar señales y ya es usada en un gran número de aplicaciones.

En esta charla presentaremos una introducción a esta técnica incluyendo los tipos de matrices aleatorias mas usadas como la representación de la transformación. Usando Matlab, se mostrarán ejemplos con señales aleatorias y en imágenes reales. También presentaremos una explicación del por qué estas matrices son apropiadas en la adquisición compresiva.

Palabras clave: Matrices aleatorias, adquisición compresiva, procesamiento de señales, Teorema de Nyquis–Shanom.

BCH-like codes from a Norm-Trace curve family

Heeralal Janwa, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Fernando L. Piñero, Department of Mathematics, University of Puerto Rico at Ponce.

In this article we describe how to find the parameters of a class of BCH like codes derived from the subfield subcodes of extended Norm–Trace (NT) codes constructed from NT curve family . With

a Gröbner basis of the ideal of the $\mathbb{F}_{q^r}$ rational points of the Norm–Trace curve we determine the dimension of the subfield subcodes or the dimension of the trace code. We also find a BCH–like bound from the minimum distance of the original supercode.

A framework for describing conceptions of slope

Rafael Martínez–Planell, Rafael Department of Mathematical Sciences, University of Puerto Rico at Mayagüez and Interamerican University at San Germán.
Courtney Nagle, The Behrend College, Penn State Erie.
Deborah Moore-Russo, College of Education, The University at Buffalo.

This paper builds on the cannon of literature on the concept of slope to introduce a framework for describing conceptions of slope. The talk starts with a brief description of previous work by Nagle and Moore-Russo in which eleven possible conceptualizations of slope are proposed. This is followed by a brief description of the main ideas of Action-Process-Object-Schema (APOS) Theory. By simultaneously considering how slope is viewed and how it is used, we brings together the ideas of APOS theory with research on slope conceptualizations to describe how students may develop an understanding of slope. The resulting framework also provides researchers a way to classify student understanding of slope and educators a guide to help design slope tasks.

Keywords: slope, APOS Theory, slope conceptualizations.

Students relating of integrals of functions of two variables and Riemann sums

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María Trigueros Gaismán, Departamento de Matemáticas, Instituto Tecnológico Autónomo de México.

Action-Process-Object-Schema Theory (APOS) is used to study students geometric understanding of partition of a rectangular domain and corresponding Riemann sum of an integral of a function of two variables. We start by giving a brief introduction to the main ideas of APOS theory. Then, a conjecture (called a genetic decomposition) of mental constructions that may be used to understand the notion of an integral of a function of two variables over a rectangle is proposed. We present the results of student interviews designed to test the conjecture. The semi-structured interviews were conducted with ten students who had just finished taking a traditional course in multivariable calculus. In this presentation we mainly consider the most basic case of a partition, that consisting of a single rectangle (the domain itself). Results show that these students had many difficulties with even the most basic mental constructions needed to relate Riemann sum and double integral. This is an important observation since some of these mental constructions are commonly assumed to be obvious to students.

Keywords: APOS Theory, integration, functions of two variables, Riemann sum.

The signed enhanced principal rank characteristic sequence

Xavier Martínez–Rivera, Department of Mathematics, Iowa State University.

The *principal minor assignment problem* asks the following question: Can we find an $n \times n$ matrix having prescribed principal minors? As a simplification of this problem, researchers associated a sequence with a symmetric (or complex Hermitian) matrix, which they defined as follows: The *enhanced principal rank characteristic sequence* (*epr-sequence*) of an $n \times n$ symmetric (or complex Hermitian) matrix B is $\ell_1\ell_2\cdots\ell_n$, where ℓ_k is $\mathtt{A}$ (respectively, $\mathtt{N}$) if all (respectively, none) the principal minors of order k are nonzero; if some (but not all) are nonzero, then $\ell_k = \mathtt{S}$.

As a refinement of the epr-sequence, the present speaker recently introduced the *signed enhanced principal rank characteristic sequence* (*sepr-sequence*) of an $n \times n$ Hermitian matrix, denoted $t_1t_2\cdots t_n$, where t_k is either $\mathtt{A}^*$, $\mathtt{A}^+$, $\mathtt{A}^-$, $\mathtt{N}$, $\mathtt{S}^*$, $\mathtt{S}^+$, or $\mathtt{S}^-$, based on the following criteria: $t_k = \mathtt{A}^*$ if B has both a positive and a negative order-k principal minor, and each order-k principal minor is nonzero. $t_k = \mathtt{A}^+$ (respectively, $t_k = \mathtt{A}^-$) if each order-k principal minor is positive (respectively, negative). $t_k = \mathtt{N}$ if each order-k principal minor is zero. $t_k = \mathtt{S}^*$ if B has each a positive, a negative, and a zero order-k principal minor. $t_k = \mathtt{S}^+$ (respectively, $t_k = \mathtt{S}^-$) if B has both a zero and a nonzero order-k principal minor, and each nonzero order-k principal minor is positive (respectively, negative).

In this talk, the epr- and sepr-sequences are introduced, and results regarding the attainability of epr- and sepr-sequences are presented. In particular, it is shown that certain subsequences cannot occur in the sepr-sequence of a Hermitian matrix.

Keywords: Signed enhanced principal rank characteristic sequence, enhanced principal rank characteristic sequence, minor, rank, Hermitian matrix.

Linear recurrences for trapezoid and rotation symmetric Boolean functions

Luis A. Medina, Department of Mathematics, University of Puerto Rico at Río Piedras.

Rotation symmetric Boolean functions are invariant under circular translation of indices. These functions have very rich cryptographic properties and have been used in different cryptosystems. It is a well-known result that sequences of exponential sums of rotation symmetric Boolean functions of degree 2 or 3 satisfy homogeneous linear recurrences. One consequence of this is that exponential sums of rotation symmetric Boolean functions of degree 2 or 3 can be computed efficiently. In this work we introduced the concept of trapezoid Boolean functions. We show that sequences of exponential sums of trapezoid Boolean functions satisfy homogeneous linear recurrences and, as a by product, that sequences of exponential sums of rotation symmetric Boolean functions also satisfy homogeneous linear recurrences (regardless of the degree).

Keywords: Rotation symmetric Boolean functions, exponential sums, linear recurrences.

Khovanov homology for almost alternating knots

Gabriel Montoya–Vega, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Juan A. Ortiz Navarro, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Although knot theory has been around since the end of the nineteenth cen- tury, it gained steam and produced some of its most exciting results within the last thirty years. In this way, being the classification the main aim of this theory, the large quantity of almost alternating knots gives rise to an important cate- gory. Following a result in Adams (1992) esteemed paper on almost alternating knots, we assume the task of finding which specific knots differ from the others satisfying an equation and the reasons why these occurs. In order to detect the knots, we established a result previously given for the span of the bracket polynomial for almost alternating knots, in terms of the Jones polynomial. The Khovanov complex, conducive to the homology, of a given knot K is generated by considering a planar projection of the knot with 2^n states, each of which consists of a collection of simple closed curves on the plane. Using the fact that Khovanov homology is a strong invariant of knots, we use Mathematica as a tool to find out some aspects that made each knot unique. Moreover, contin- uing with our motivation of exploring new areas in this theory, we introduce the surfaces for knots. Finally, we show the relevant knots and the facts that, in contrast with the others, make them not hold the equation, giving special attention to the polynomial generated by the Khovanov complex.

Keywords: Alternating knots, bracket polynomial, Khovanov homology.

Fast Arithmetic in $GF(p^2)$ and $GF(p^3)$ for a Certain Family of Primes

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Dorothy Bollman, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The most commonly used finite fields in applications are the binary fields $GF(2^m)$ and the prime fields $GF(p)$. For binary fields, arithmetic in the ground field is generally efficient because it corresponds to computer arithmetic, but reduction modulo an irreducible polynomial over the ground field can be time consuming. Such a reduction is not needed in the case of prime fields, but reduction modulo a prime can be a bottleneck, particularly when the prime is very large. These problems can be mitigated by choosing a prime p and an appropriately chosen irreducible polynomial that is irreducible over $GF(p)$ such that $GF(p^m)$ has approximately the same size as the given binary or prime field and in which reductions in both the ground and extension fields is efficient.

We give conditions under which binomials $x^m - \omega$, $m = 2, 3$ and $\omega = 2, 3$, are irreducible modulo primes of the form $2^a - 2^b - 1$. We show how reductions in the resulting extension and ground fields

can then be carried out using only shifts and additions in $GF(p)$, without either multiplications or divisions.

Keywords: optimal extension fields, irreducible binomials.

A numerical scheme that avoids the repulsion property in nonlinear elasticity

Pablo V. Negrón–Marrero, Department of Mathematics, University of Puerto Rico at Humacao.

For problems of the Calculus of Variations that exhibit the Lavrentiev Phenomenon, it is known that the repulsion property holds. That is, if one approximates the global minimizer in these problems by smooth functions, then the approximate energies will blow up. In SIDIM 2014 we showed that the repulsion property holds in the context of nonlinear elasticity, in particular for problems in which the minimizers exhibit cavitation. In this talk we propose a numerical scheme that circumvents or works around the repulsion property for problems in elasticity. We discuss a general result for the convergence of this scheme and for the case of a spherical body, we prove the convergence to the actual cavitating minimizer.

Keywords: nonlinear elasticity, Lavrentiev phenomenon, cavitation.

On a family of finite fields for fast FPGA implementations of Elliptic Curve point multiplication

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Einstein Morales, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Dorothy Bollman, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

We consider elliptic curve point multiplication over fields of the form $\mathbb{F}_{p^3}$ that are variations of optimal extension fields, with p a prime of the form $2^n - 2^i - 1$. These fields are particularly well suited for implementations of elliptic curve point multiplication on field programmable gate arrays (FPGAs). Such an implementation can outperform FPGA implementations for NIST recommended fields that have approximately the same number of bits. We illustrate these ideas with an FPGA implementation of point multiplication over the 162 bit field $\mathbb{F}_{p^3}$ where $p = 2^{54} - 33$ and we compare its performance to that of a known implementation for the NIST recommended field $\mathbb{F}_{2^{163}}$.

Keywords: pseudo-Mersenne prime, optimal extension field, elliptic curve cryptography, point multiplication, FPGA.

Mathematical Model of Stress Variation on the Hypothalamic–Pituitary–Adrenal Axis (HPA) and its repercussion on hormones levels (Cortisol, CRH, ACTH and GR)

Alexis Ortiz Vega, Department of Mathematics–Physics, University of Puerto Rico at Cayey.
Mayteé Cruz–Aponte, Department of Mathematics–Physics, University of Puerto Rico at Cayey.

Stress caused by psychosocial factors creates a hormonal response in our body. This hormonal response is caused by activation in the hypothalamic–pituitary–adrenal axis (HPA axis). Psychosocial factors cause different levels of stress (recent, chronic or daily); these types of stress develop in different periods of time. Although, in some cases a person may present multiple types of stress at the same time due to the variability of psychosocial factors. The purpose of this research is to simulate the hormonal relationship of the HPA axis and the stress caused by psychosocial factors. A linear function for stress was developed and incorporated into the non-linear ODE system of the cortisol network, to observe how the HPA axis behaves when it is affected by stress impulses over a period of time. As a result, we found that the stress caused by psychosocial factors is proportional to the levels of the hormones in the HPA axis (cortisol, ACTH, CRH and GR) affected by these impulses.

An estimate of the basic reproduction number R_0 for the 2015–2016 Zika virus outbreak in Puerto Rico

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In recent years, vector-borne diseases are taking serious attention from researchers and health specialist across the world. The emergence of vector-borne diseases, such as Chikungunya and Zika, coupled with outbreaks of both diseases in the Americas are of great interest to the scientific community since there is still very much to learn about their transmission, risks and effects.

The Zika virus (ZIKV) is transmitted by infected females Aedes aegypti mosquitoes, but there is also confirmed evidence that it can be transmitted directly (human to human) by sexual contacts and from mother to fetus. The apparent effects in the neurological system through the Guillan-Barré syndrome and the neonate microcephaly are of great concern. In Puerto Rico, 64% of the confirmed cases are females, and since the Zika disease is usually asymptomatic, pregnant women may not even know that they have the virus. At the end of December 2016, the Puerto Rico Department of Health estimated 36,500 cases.

In this work, we focus on the 2015-2016 Zika virus (ZIKV) outbreak in Puerto Rico and use the data of confirmed by laboratory Zika cases obtained from the weekly reports published by the Puerto Rico Department of Health. To analyze the behavior of the Zika virus in Puerto Rico, we consider a mathematical model taking into account direct (through vector) and sexual transmission. Using the data and the epidemic model, we estimate the initial exponential growth rate, (defined as the force of infection), using different statistical methods, in order to estimate the basic

reproduction number (R_0) of the Zika epidemic in Puerto Rico.

Keywords: Zika virus, estimation, data analysis, statistical methods, Zika epidemic model, basic reproduction number, force of infection, sexual transmission.

Two mathematical approaches to study phosphorus dynamics in Laguna Cartagena, Puerto Rico

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Laguna Cartagena (LC), a wetland in Lajas, Puerto Rico, has been negatively impacted by nutrients, mainly phosphorus run-off from agricultural activities until the end of sugar cane cultivation in the late 1900s. This led to phosphurus concentration remains high at hypereutrophic state that was irremediable even after a 5-fold reduction in source water nutrient concentration. Grey Clustering Method (GCM) was used to classify LC's eutrophic state by applying the International and Chinese trophic standards and using two parameters, total phosphorous (TP) and total nitrogen (TN). Mean TP and TN from LC consolidated bottom substrate and flocculence samples were used to classify LC. To address whether LC can recover, soluble reactive phosphorus (SRP) and TP from LC inlet, outlet and center water samples were used to model the input and loss of phosphorus in LC and determine whether an equilibrium point exists. GCM analysis classified LC as a eutrophic wetland using the International standard and hypereutrophic using the Chinese standard. Trophic state classification did not vary with use of consolidated bottom substrate versus flocculence samples. A differential equation model showed that SRP and TP levels within LC were higher than levels of SRP and TP entering LC, which could be caused by a nutrient recycling process within LC that may predict failure of remediation efforts. An equilibrium point was found at the eutrophic level, which means that even if there is a reduction in phosphorus input, there will not be a change in LC's eutrophic state.

Keywords: phosphorus dynamics, total phosphorus, soluble reactive phosphorus, total nitrogen, differential equation, eutrophication, Grey Clustering Method.

Linear codes related to Grassmannians as Tanner codes

Fernando L. Piñero, Department of Mathematics, University of Puerto Rico at Ponce.

The Grassmannian is one of the classical algebraic varieties. The linear code generated from the $\mathbb{F}_q$–rational points is one of the fundamental techniques to study any algebraic variety. The Grassmann code is the linear code related to the Grassmannian. Several interesting properties of the

Grassmannian are preserved in the Grassmann codes. In this talk, we use Tanner codes to study Grassmann codes and related codes . In this way we can study Grassmann codes, Schubert codesm, Schubert union codes and other codes related to the Grassmannian. In fact, we solve a longstanding conjecture on the minimum distance of Schubert union codes.

Keywords: Grassmann codes, Schubert codes, Linear Codes, Tanner codes.

Khovanov homology for $(3, k)$–torus knots

Yolima A. Rocha Fontalvo, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Juan A. Ortiz Navarro, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

In this talk we will study the construction of Khovanov homology for $(3, k)$–torus knots by using combinatorial topology and skein theory, identifying common characteristics of Khovanov Bracket for $(3, k)$–torus knots. The r–th homology, $\mathcal{H}^r$, of the complex $\mathcal{C}$ is calculated explicitly for $r = 0, 1, 2k - 1$ and $2k$, it allows to obtain some exponents of the variables q and t in the graded Poincaré polynomial of the complex $\mathcal{C}$, which is called the Khovanov bracket.

Keywords: Khovanov homology, Khovanov Bracket, torus knot.

Low Stretch Spanning Trees

Gustavo Gratacós, Department of Computer Science, University of Puerto Rico at Río Piedras.
Alberto J. Ruiz Sandoval, Department of Computer Science, University of Puerto Rico at Río Piedras.

Current algorithms to find low stretch spanning trees of a graph are infeasible to implement. In addition, many implementations are tailored to a small set of specific problems. We are working on the design and implementation of an algorithm that achieves a partition of a graph into dense clusters in a pragmatic manner. Our algorithm labels each vertex of the graph with a number that represents their corresponding neighbourhood. Afterwards, it sorts the vertices so that neighbourhoods are coupled together. The algorithm then scans the graph in this order, partitioning where appropriate. From this, we choose suitable edges to include in our low stretch spanning tree. This has resulted in some promising results for dense graphs. We are currently implementing similar algorithms using this heuristic in order to yield better results on sparse graphs.

A Simplicity Criterion for Rank One Local Systems

Luis E. Saumell, Department of Mathematics, University of Notre Dame, Notre Dame, Indiana.

In this talk we will talk about rank one local systems on a topological space. The emphasis will be on local systems on complements of hyperplane arrangements. We will identify these local systems with regular holonomic D-Modules via the Riemann-Hilbert Correspondence. The category of Holonomic D-Modules is Artinian and so we can speak of the length of these objects. An object is said to be simple if it has length one. The main result in the talk is that we present a criterion for simplicity and in the case of hyperplane arrangements, this criterion is combinatorial. We will compute several examples.

Acknowledgements: This is joint work with Nero Budur (Math Department, KU Leuven, Belgium), Yongqiang Liu (Math Department, KU Leuven, Belgium), and Botong Wang (Math Department, University of Wisconsin– Madison).

Keywords: Local Systems, D-Modules, Perverse Sheaves, Hyperplane Arrangements.

Mathematical model and analysis of micro vibration of Cosserat plates

Lev Steinberg, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Roman Kvasov, Department of Mathematics, University of Puerto Rico at Aguadilla.

In this talk we present our recently developed mathematical model for the dynamics of Cosserat elastic plates and our preliminary numerical results of the plate vibration. The recent extension is based on the variational principle for elastodynamics and includes modeling for the case of different shapes and orientations of micro elements incorporated into the Cosserat plates. The generalized dynamic model has been tested using numerical computations of the plate free vibration. The appearance of the additional high frequencies of micro vibrations depending on the orientation of micro elements has been detected and will be discussed. Also, the model has been tested in comparison with three–dimensional Cosserat elastodynamics. The preliminary computations of eigenfrequencies show the high agreement with the exact values.

Keywords: variational principle, Cosserat plate vibration, frequencies of micro vibration.

Bayesian approach for a semiparametric mixed model with beta distribution

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Black Sigatoka is a leaf-spot disease of banana plants that it is measured using the Stover–Gauhl scale. Typically, several leaves are measured on this scale to compute a severity index (SI) that indicates the amount of damage in the plant. The SI is often expressed in a 0–1 scale. Existing approaches use semiparametric models with normal response after a logit transformation to analyze this type of data. However, this approach could be inefficient and inappropriate due to the asymmetric distribution of the response even after transformation. We propose a Bayesian semiparametric beta regression to model the severity disease progress. The proposed model allows the response variable to follow a beta distribution which adjusts better to the distribution of the severity index. The Bayesian approach facilitates the comparison of curves across time without having to adjust for multiple comparisons, a limitation of the frequentist approach. The nonparametric terms in the proposed model are handled using b–splines basis functions. We applied the proposed models Black Sigatoka disease on banana crop data from Puerto Rico. The proposed model is fitted in JAGS via R2jags package called from R 3.3.2 using MCMC. Further research work will focus on the study of the statistical properties of the proposed model and its comparison with existing approaches.

Keywords: MCMC, b-splines, JAGS.

Application of Grey Clustering Method to Assess Eutrophication in Six Colombian Wetlands

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Wetlands are invaluable natural resources, threatened among other things by the surplus of nutrients which trigger a process known as eutrophication. Assessment of eutrophication is a first step in wetland restoration, hence, a methodology to classify by level of eutrophication is needed as current systems use nutrient-specific indexes without consideration of nutrient interactions.

The Grey Clustering Method (GCM) consists of an iterative algorithm for classification of n-objects into one of s-classifications, combining the measurements of m-parameters of each object. In the present case of study, this method assumes that there are n-objects (wetlands), m- indexes (parameters) and s-different grey classifications (trophic categories). The goal of this research is to classify each object, using the indexes of interest, into one grey classification (trophic state).

To develop this research, published data from six Colombian wetlands located in Bogotá, Colombia were used. We used GCM and two standards (international and Chinese) with two parameters

(total phosphorus and total nitrogen) to assess the eutrophication level of the six following Colombian wetlands: Tibanica, Capellana, De Techo, La Vaca, La Conejera and Laguna Fquene.

Despite of differences in both standards, similar results were obtained after using GCM for classification of trophic state of the six Colombian wetlands. Using either standard, Laguna Fquene was classified as a eutrophic wetland, while the others wetlands were classified as eutrophic using the international standard and hypertrophic using the Chinese standard. Also, according to Grey Scores (GS), 5 out of 6 wetlands have the same level of eutrophication.

Keywords: eutrophication, total phosphorus, total nitrogen, Grey Clustering Method.

Modelling The Impact of Zika Virus Epidemic with Vaccination and Optimal Control Strategies

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Zika virus (ZIKV) is a vector-borne disease that has rapidly spread during the year 2016 in more than 50 countries around the world. The virus can cause severe birth defects and brain damage in babies if a woman is infected during pregnancy. As an intervention for controlling the spread of the disease we study the cost of a vaccination campaign as well as different optimal control strategies of preventing Zika infections in the near future. Although there is no formal vaccine for ZIKV, The National Institute of Allergy and Infectious Diseases part of the National Institutes of Health has launched a vaccine trial at the beginning of August 2016 to control ZIKV transmission. Thus, in this work, we formulate a vaccination model for Zika virus including direct transmission and optimal control strategies. We calculate the basic reproduction number of the model to analyze the impact of vaccination including, perfect and imperfect vaccination. Pontryagin's maximum principle is used to determine the necessary conditions for optimal control of ZIKV. A sensitivity analysis of the basic reproduction number is also performed to measure the most influential parameters to determine future control strategies. We illustrate several numerical examples of the vaccination model to validate our theoretical results under optimal control strategies.

Keywords: Zika Virus, Vaccination, Optimal Control, Pontryagin's Maximum Principle, Epidemic Model.

2 error binary correcting codes trough non-linear algebra

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he fundamental problem in communication is the fact that as we increase the amount of information we want to send through a channel the risk of the information being altered increases. The purpose of error correcting codes is to recover the information that was lost in the channel. Linear codes in

general can be defined by an $(n-k) \times n$ parity check matrix H where n is the block length of the codewords and k is the rank of the codes generator matrix. We study the application of nonlinear functions (focusing on the Gold and Kasami sequences) on the parity check matrix so that we can increase the minimum distance of a code and hence its error correcting capacity. In particular we utilize Hamming Codes, a family of one error correcting codes, and the properties of their parity check matrix as our base for the creation of two error correcting codes.

The application of the non-linear function creates a new parity check matrix that in turn defines a new code. We build a system of equations based on the properties of this new matrix and study parameters that would define a two error correcting code. In general, nonlinear functions defined over finite fields have very important applications in error correcting codes. The Gold and Kasami functions are related to 2–error correcting codes of length $2^s - 1$ as studied by Van Lint, Wilson, Janwa and others. We use only the properties of the Gold and Kasami functions and their domains, as well as multivariate polynomials trough which we build systems of equations in order to determine parameters that help us identify two error correcting codes.

Acknowledgements: This investigation is possible due to the efforts of Professor Moises Delgado and the Department of Mathematics at the University of Puerto Rico at Cayey.

Keywords: codes, code theory, error correction, non-linear algebra.

The quasi-linear Venttsel' problem on the Koch snowflake domain

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We will discuss recent results concerning the well-posedness and regularity theory of a differential equation with the so called Venttsel' boundary conditions, on the Koch snowflake domain.

4 Carteles / Posters

(In alphabetical order using the last name of the presenter.)

Application of Almost Perfect Non-lineal functions in the Design of Cryptographic Systems

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The use of cryptographic security systems to transfer information became very important during the last years. One of the most used security systems is the Data Encryption Standard (DES). DES has been extensively analyzed in order to understand its defensive and weakness properties. Many attacks has been treated in order to break this system however, until the differential cryptanalysis, none of them has been proved to be successful. The differential cryptanalysis is a probabilistic attack that is extremely effective against the DES-like ciphers. Nyberg and Knudsen proposed a system that is probable secure against differential cryptanalysis. We take Nyberg and Knudsens idea of using an almost perfect nonlinear function (APN) as the main component of the system, to create a new DES-like cipher that is resistant against differential cryptanalysis. APN functions are polynomial functions with optimal nonlinear properties, defined over finite fields. Finally, we compare our system security with the Data Encryption Standards and we also improve the upper bound of the success probability of a differential, with respect to previous results.

Stability of Control System of Intracellular Iron Homeostasis: A Mathematical Proof

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Iron is a metal essential for cellular metabolism. Excess or lack of iron can cause serious health conditions. To deal with these difficulties, the intracellular levels of iron are tightly constrained by a complex control network of proteins. Recently, Chifman et al. developed and validated a mathematical model in the form of five differential equations, of the core control system of intracellular iron homeostasis in normal breast epithelial cells. Their work was motivated by the fact that intracellular iron homeostasis can play a role in the pathogenesis of breast cancer. For any choice of parameters, their dynamical system has a unique equilibrium, and Chifman et al.s simulations suggest it is globally stable. Here we introduce a biologically reasonable simplification of the Chifman model. For this valid approximation, we show that it has a unique steady state and that it is locally asymptotically stable. We also give evidence that this model seems to approach global stability by using a geometric analysis. This reduced version gives us an insight on how the original model behaves.

Acknowledgements: We would like to thank our mentor, Dr. Anne Shiu. We would also like to thank Robert Williams for his assistance and Dr. Paul Lindhal for helping with the biological aspects of the research. This research was conducted as part of the NSF-funded REU program in Mathematics at Texas A&M University (DMS-1460766), Summer 2016.

Keywords: iron, global stability, mathematical modelling.

Recognition of pollen–bearing bees from video using visual feature classification

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In this communication, we present a comparison between features descriptors of images for the recognition of pollen bearing honey bees using video captured at the entrance of their hives. The goal is to select the best feature and the classifier for scalability purposes. Several visual features were considered to describe the bees, such as RGB Image, a proposed yellowness attribute that detects pollen color and its post-processing with spatial filtering. Histograms from the previous features where also considered. Several classifiers, such as KNN, SVM and Naive Bayes, were applied to detect the presence of pollen based on these features. An annotated dataset was created containing both pollen bearing and non pollen bearing bees. It contains 600 images of bees extracted from a 40s video. Evaluation was performed using stratified cross validation. Based on the size of the descriptors files and the accuracy in the classification, the following insights were obtained. Experimental results showed that best results (92.54%) were obtained using flattened image features on 150x100 pixel patches. This keeps the spatial information, but produces large dimensional features (15000 dimension in this case). Histogram type features produce much more compact features (20 dimensions were used) to be used for larger-scale analysis and indexing. Its performance was less, but was improved by combining the yellowness attribute and further filtering by laplacian of gaussian which enhances the blob structures of the pollen balls. In the case of SVM classifier, the increase observed was from 63.96% to 86.40%. These results confirm that both color and spatial information capturing position and shape of the pollen balls are important information for the recognition and provide strategies for different accuracy/compactness trade-off choices.

In the future, it is planned to expand this work to larger–scale datasets, by using semi-supervised and active learning to take advantage of the large amount unannotated video data available.

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Parameter identifiability on a mathematical model for prostate cancer treatment

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Although it is not still clear what causes prostate cancer, it has been known for a long time that its growth is stimulated by androgens, or male sexual hormones. However, one of the principal screening methods for prostate cancer consists of measuring the Prostate Specific Antigen (PSA), which is a protein produced by prostate cells. Using a mathematical model by Hirata et al., we tried to predict PSA levels outcomes for individual patients by doing data fitting, but most of the results were quite unreliable and illogical (for example, negative or extremely high PSA values). As numerical optimization algorithms usually find difficulties when trying to estimate and obtain unique values for parameters of a non-identifiable model, a parameter identifiability analysis was performed. By using what is known as the transfer function, we were able to show that the Hirata model was non–identifiable under a specific parameter space, therefore justifying the presence of aleatory parameter values on the data fitting.

Acknowledgements: We thank National Science Foundation for funding this research through the Mathematical Biosciences Institute.

Keywords: prostate cancer, PSA, data fitting, parameter identifiability, transfer function.

Modelling Vaccination Strategies of Full and Reduced Doses for a Yellow Fever Epidemic in Angola

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Yellow fever is a viral hemorrhagic disease that kills approximately 30,000 individuals worldwide every year. It is transmitted by the female mosquito from the *Aedes* genus and the majority of cases occur in countries located in Africa, Central America and South America. Around 1930, researchers developed the 17D yellow fever vaccine to attempt to control the great surge of yellow fever outbreaks in prior decades. Although this effective vaccine provides life-long immunity, the extensive production time and lack of worldwide manufacturers make it difficult to alter the normal yearly vaccine production pattern. In 2015, amid a yellow fever outbreak in Angola, international public health agencies began to take notice that the worldwide 17D vaccine supply would not be sufficient to control the ongoing epidemic. In response, authorities from the World Health

Organization proposed a new vaccination method: using a portion of the full dose (one fifth) to provide short-term immunization to more individuals. In this work, an SVIR mathematical epidemic model is developed to evaluate the pros and cons of the full dose and portioned (1/5th) vaccination strategies under different epidemic and vaccination scenarios in Angola. We calculate the basic reproductive number (R_0) of yellow fever and performed numerical simulations to the aforementioned model. We hope to further study the disease's spread in the region of Angola and determine the ideal vaccination strategy to employ under an epidemic.

Keywords: yellow fever, epidemic, vaccination, fractional dose, mathematical modeling, SVIR.

Mathematical modeling of Quarantine intensity and application time for the 2002-2003 SARS outbreak in China

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The severe acute respiratory syndrome (SARS) is a serious form of pneumonia related to a virus that was first identified in late 2002. We recovered data from the outbreak in China to project the impact of a control measure when it is introduced at different times in an epidemic. At the same time, we also vary the intensity of quarantine to view the effect it would have with the data recollected. Our modeling approach consists of a three (ODE) model; the first equation correspond to the basic composition of a SEIR model. The second equation includes the quarantine and the final one, which is similar to the second equation, we can manipulate the intensity of quarantine and the time it is applied. Our preliminary results show that quarantine was only as effective when it was applied at a certain interval of time. A project like this one can be used to study other diseases, not just SARS, to be able to understand the importance of quarantine and minimize the resources needed to contain epidemics.

Keywords: time of application, quarantine, SARS.

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