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1   Charlas Plenarias / Plenary Talks

Drift vs Shift: Decoupling Trends and Changepoint Analysis

David Matteson, Cornell University.

Distinguishing between global or macro patterns and local or micro fluctuations helps summarize the evolution of complex non-stationary dynamic systems. Herein, we focus on making distinctions between drift and shifts. Drift describes the micro-level evolution of a process. This may appear as variation about gradual trends. In contrast, shifts refer to discontinuities, rapid changes, or major breaks in trend. These represent macro-level changes in a process. Both trends and shifts might be mechanistically or stochastically generated and/or modeled. However, the underlying causes of shifts are typically different from those of drift. While understanding such differences is a prime objective, this first requires distinguishing shifts from drift.

We introduce a new approach for decoupling trends (drift) and changepoints (shifts) in time series. Our locally adaptive model-based approach for robustly decoupling combines Bayesian trend filtering and machine learning based regularization. An over-parameterized Bayesian dynamic linear model (DLM) is first applied to characterize drift. Then a weighted penalized likelihood estimator is paired with the estimated DLM posterior distribution to identify shifts. We show how Bayesian DLMs specified with so-called shrinkage priors can provide smooth estimates of underlying trends in the presence of complex noise components. However, their inability to shrink exactly to zero inhibits direct changepoint detection. In contrast, penalized likelihood methods are highly effective in locating changepoints. However, they require data with simple patterns in both signal and noise. The proposed decoupling approach combines the strengths of both, i.e. the flexibility of Bayesian DLMs with the hard thresholding property of penalized likelihood estimators, to provide changepoint analysis in the complex, modern settings. The proposed framework is outlier robust and can identify a variety of changes, including in mean and slope. It is also easily extended for analysis of parameter changes in time-varying parameter models like dynamic regressions. We illustrate the flexibility and contrast the performance and robustness of our approach with several alternative methods across a wide range of simulations and application examples.

Jupyter: tools for open interactive computing across disciplines

Fernando Perez, UC Berkeley.

Jupyter is widely used today across disciplines, from astronomy to microbiology. I will discuss some of the project's history and architecture, focusing on the changing landscape of open science and the role of open source software in research and education.

Espacios simétricos y retículas

Sebastian Hurtado, University of Chicago.

Los espacios simétricos y sus retículas son objetos de fundamental importancia en Topología, Geometría, Teoría de números, entre otras áreas. La charla será una breve introducción a estos objetos y su relevancia en estas areas. Si el tiempo lo permite, se discutirán nuevos resultados sobre estos objetos obtenidos en colaboración con Mikolaj Fraczyk y Jean Raimbault.

2 Sesiones Temáticas/ Thematic Sessions

2.1 Data Science in Science

Catalysts help make chemical reactions go faster and their development impact areas such as energy, the environment, biotechnology, and drug design. Seeking to harness computational data science tools to perform data-driven discovery of new catalysts, a collaborative team is assembled with the complementary expertise in catalysts, materials science, biophysics, computational modelling, statistics, and data science. How a reaction is accelerated depends on the dynamic changes in the structure and shape of a catalyst and its associated chemical reactants (a catalytic system). The goal of this project is to explore, describe, and quantify the dynamic structures of enzyme and nanoparticle catalysts at the atomic level. Recent advances in microscopy and spectroscopy now make it possible to measure dynamics in high temporal resolution. Our team combines recent advances in data science with these new experimental tools to extract features that describe the dynamic behavior of catalytic systems. To achieve our objectives, we must tackle the severely degraded signal-to-noise ratios of the data. Specifically, the poor signal-to-noise associated with high temporal resolution makes it challenging to determine the position and intensity of atomic columns in materials undergoing structural dynamics or the latent states of enzymes during spectroscopy experiments.

To address these challenges, we propose a denoising deep learning approach, a noise-robust processing approach based on blob detection, and mixture representations to summarize biomedical data. Organized by Roberto Rivera.

Data Science in Science

Roberto Rivera Santiago, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Catalysts help make chemical reactions go faster and their development impact areas such as energy, the environment, biotechnology, and drug design. Seeking to harness computational data science tools to perform data-driven discovery of new catalysts, a collaborative team is assembled with the complementary expertise in catalysts, materials science, biophysics, computational modelling, statistics, and data science. How a reaction is accelerated depends on the dynamic changes in the structure and shape of a catalyst and its associated chemical reactants (a catalytic system). In this talk we discuss the experiments (Transmission Electron Microscopy and single molecule FRET

microscopy) , challenges to process such data, briefly discuss some approaches our team pursues, and suggest future research directions.

Adaptive denoising: generalizing pre-trained denoisers to out-of-distribution data

Sreyas Mohan, New York University.

Deep convolutional neural networks (CNNs) for image denoising are usually trained on large datasets. These models achieve the current state of the art, but they have difficulties generalizing when applied to data that deviate from the training distribution. In this talk, we will introduce "GainTuning", in which CNN models pre-trained on large datasets are adaptively and selectively adjusted for individual test images. GainTuning improves state-of-the-art CNNs on standard image-denoising benchmarks, boosting their denoising performance on nearly every image in a held-out test set. These adaptive improvements are even more substantial for test images differing systematically from the training data, either in noise level or image type. We illustrate the potential of adaptive denoising in a scientific application, in which a CNN is trained on synthetic data, and tested on real transmission-electron-microscope images. In contrast to the existing methodology, GainTuning is able to faithfully reconstruct the structure of catalytic nanoparticles from these data at extremely low signal-to-noise ratios.

Deep learning based 3D feature extraction in transmission electron microscopy

Adria Marcos Morales, Interdisciplinary Higher Education Centre, UPC.

The analysis of nanoparticles is a booming research field in material science, with applications such as catalysis, medicine or optoelectronics. The physical and chemical properties of these particles rely on their exact 3D structure, and multiple approaches have been developed to extract this information, being scanning transmission electron microscopy (S/TEM) the most accurate at the moment. It performs at the atomic scale but requires the acquisition of at least two different images from different zone axes to retrieve the 3D structure, demanding stable systems. In addition, the short exposure time in the image acquisition produces highly noisy images. Convolutional neural networks (CNNs) are capable of extracting the morphology of a nanoparticle from a single image by determining the number of aligned atoms per column observed, even in such noisy images. In this talk we will describe this approach to structure reconstruction on cerium dioxide (ceria, Cr02) nanoparticles and the different techniques applied for that purpose.

Recording atomic column positions and intensities via Blob Detection in noise-degraded TEM frames

Yuchen Xu, Cornell University.

Capable of recording dynamic events at the atomic level (so-called fluxional behavior), in situ transmission electron microscopy (TEM) has limitations when quantifying the systems' atomic rearrangements as high temporal resolutions corrupt the quality of the frames with severely degraded signal-to-noise ratios. Along with some tailored subsequent refinements, we propose a relatively noise-robust routine based on blob detection, which has been established among the researchers of computer vision while remains fresh in this new field, to locate the atomic columns and measure corresponding integrated intensities. Compared to most of the conventional and renowned image analysis procedures in the community of material science, we show that our blob detection algorithm outperforms when handling noisy TEM image-series, with evidence including comparably better precision and consistency on both simulated and experimental images.

Mixture representations for likelihood ratio ordered distributions

Michael Jauch, Cornell University.

In many statistical applications, subject matter knowledge or theoretical considerations suggest that two distributions should satisfy a stochastic order, with samples from one distribution tending to be larger than those from the other. In these situations, incorporating stochastic order constraints can lead to improved inferences. This talk will introduce mixture representations for distributions satisfying a likelihood ratio order. To illustrate the practical value of the mixture representations, I'll address the problem of density estimation for likelihood ratio ordered distributions. In particular, I'll propose a nonparametric Bayesian solution which takes advantage of the mixture representations. The prior distribution is constructed from Dirichlet process mixtures and has large support on the space of pairs of densities satisfying the monotone ratio constraint. I'll demonstrate the approach in a biomedical application.

2.2 Factorization and graphs

In 1988 Beck developed the notion of a zero-divisor graph on commutative rings. Basically, the idea was to study the graph of a nonzero element given by zero-divisor as vertices and two vertices were connected if the zero-divisors appears in a factorization of the element. With this notion, many properties of the commutative rings can be interpreted by the zero-divisor graph of its elements. There are many papers in this subject, as a matter of fact more than 50 papers just between 2000-2013.

The notion of zero-divisor graphs motivated Coykendall and Maney, in 2007, to develop the notion of the irreducible-divisor graphs. The same concept was studied by Axtell and Stickles in 2008, and further developed with Baeth in 2011. They were able to characterize types of integral domains with weaker factorization properties than a unique factorization domain. The question

that rises is whether we can do this with other types of elements.

In 2013, Mooney used the theory of generalized factorizations or -factorizations to extend the notion to -irreducible -divisor graph on integral domain and commutative rings. Since 2013, Ortiz and his students have been working on several special cases and try to extend such notion to any possible factor.

These talks would give an idea of the theories involved, examples, and show some new directions in which the theory is moving towards. Aside of creating more mathematics, this theory is being used to characterize structures and solve problems in graph theory that with the algebraic component would be very hard or impossible. Organized by Reyes M Ortiz-Albino.

Resumen de algunos datos sobre τ-factorizaciones

Reyes M. Ortiz Albino, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

La noción de las τ-factorizaciones o τ-productos sobre dominios con integridad surge como un tipo de factorizaciones generalizadas definidas por Anderson y Frazier, en el 2006. Esta nueva noción sirve como base para el estudio de las distintas tipo de factorizaciones de una misma estructura o simplemente concentrar el estudio de factorizaciones de un subconjunto de elementos de la estructura. Entre los resultados principales de Anderson y Frazier, el mas que resalta es la caracterización de tres tipos de relaciones τ que garantizan propiedades de factorizaciones. Tales propiedades son naturales en los productos usuales y hacen que propiedades de la estructura de factorizaciones del dominio con integridad se hereden a las τ-factorizaciones.

Uno de los ejemplos que presentaron fue el concepto de los $\tau_{(n)}$-productos que solo permite que dos elementos $x, y \in \mathbb{Z} - \{0, \pm 1\}$ se puedan multiplicar si y solo si $n \mid (x - y)$. Similarmente, una $\tau_{(n)}$-factorización de $x \in \mathbb{Z} - \{0, \pm 1\}$, es una expresión de la forma $x = \pm x_1 * x_2 * * * x_n$ donde cada $x_i \in \mathbb{Z} - \{0, \pm 1\}$ y para todo $i \neq j$ $n \mid (x_i - x_j)$. Estas factorizaciones han sido estudiadas por Frazier, Hamon, Ortiz, Juett, Florescu, Mooney y varios estudiantes de Ortiz.

Esta presentación proveerá las nociones básicas de la teoría de factorizaciones generalizadas para las charlas de los grafos de τ-factores de Mooney, Romero y Jiménez. Esta idea es muy análoga a la de Coykendall en el 2007, Axtell en el 2011, Mooney en el 2013 y Lopez 2020 han presentado. Se prevee que esta teoría pueda ser de ayuda a resolver problemas con alta complejidad y caracterizar grafos con propiedades únicas.

Keywords: generalized factorizations, factorizations

On gracefully labeling zero-divisor graphs

Christopher Park Mooney, Department of Mathematics, Statistics and Computer Science, University of Wisconsin – Stout.

In this talk, we present some recent research on graceful labeling zero-divisor graphs of a commutative ring R with $1 \neq 0$. The zero-divisor graph $\Gamma(R)$ is the undirected graph whose vertex set is comprised of the nonzero, zero-divisors of R and whose edge set is given by the relationship: distinct

vertices x and y are adjacent in $\Gamma(R)$ if and only if $xy = 0$. This graph comes up naturally when thinking about factorization properties in commutative rings with zero-divisors. Classically, the question was about the coloring number of zero-divisor graphs, but here we look at a different type of labeling called graceful labeling. We are able to find infinite classes of rings which are graceful and infinite classes of rings which are not graceful. We then turn our attention to the question of which rings admit graceful zero-divisor graphs beginning with all zero-divisor graphs on up to 14 vertices. We conclude by posing a few questions and thinking about potential for future research.

Keywords: factorization, zero-divisor graphs, graph labeling, graceful labeling

Grafos de factores en dominios con integridad

Offir N. Romero Castro, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.
Reyes M. Ortiz Albino, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

Anderson y Frazier (2006) definieron la teoría de τ-factorizaciones o de factorizaciones generalizadas utilizando una relación simétrica τ sobre $D^{\#}$, el conjunto de elementos distintos de cero y de unidades de un dominio integral D. La idea se puede interpretar como el estudio del producto de elementos que se relacionan con respecto a τ. Este concepto generalizó muchos casos de factorizaciones previamente estudiadas como: factorizaciones primas, factorizaciones en elementos irreducibles, factorizaciones comaximales, entre otras.

Sea $\mathcal{P}(D^{\#})$ el conjunto potencia de $D^{\#}$, $\alpha \in \mathcal{P}(D^{\#})$ y τ una relación simétrica sobre $D^{\#}$. Este trabajo considera la subrelación $\tau^{\alpha} = \{(a,b) : a,b \in \alpha \text{ y } a\tau b\}$ de τ y presenta algunas de sus características y propiedades. Se define el grafo de factores de un elemento en $D^{\#}$ como una generalización del grafo de factores irreducibles de Coykendall y Maney, del grafo de τ-factores τ-irreducibles y del grafo de α-β-divisores de Mooney. Se muestran algunos ejemplos y propiedades de los grafos de factores, así como las implicaciones de la relación τ^{α} en sus subgrafos.

Palabras clave: grafos de factores, factorizaciones generalizadas, elementos asociados

Relación amiga, grafos y su generalización

Julián A. Jiménez Franco, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.
Reyes M. Ortiz Albino, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

Anderson y Frazier en 2006 definieron el concepto de factorizaciones generalizadas sobre dominios enteros. Para esto, los autores restringieron la operación multiplicativa de manera que solo permiten multiplicar los elementos que estén relacionados con respecto a una relación simétrica τ. Esto abre puertas a re-estudiar la teoría de factorizaciones sobre dominios y temas similares desde muchos

puntos de vista. Por ejemplo, Hamon, Ortiz, entre otros, han estudiado las factorizaciones de enteros que se relacionan con respecto a la relación de equivalencia módulo n. Aún Ortiz continúa estudiando con su grupo de investigación desde el punto de vista de teoría de números, teoría de grafos, entre otros. Este trabajo considera otra relación de equivalencia sobre $\mathbb{Z}^{\#} = \mathbb{Z} - \{0, \pm 1\}$ llamada relación amiga.

La relación amiga fue definida en la XXIII Olimpiada Colombiana de Matemática y se denota por $R_2 = \{(a,b) \in \mathbb{Z}^{\#} \times \mathbb{Z}^{\#} \mid \sqrt{ab} \in \mathbb{Z}\}$. Es decir, $(a,b) \in R_2$, si $\sqrt{ab} \in \mathbb{Z}$. Se estudian algunos conceptos de Anderson y Fraizer aplicados a la relación amiga. Entre ellos τ-factorización, elemento τ-irreducible, τ-primo y el grafo de τ-factores τ-irreducibles. Además, se presenta la relación $R_{\frac{m}{n}}$ como una generalización de R_2. Se muestran algunos ejemplos y resultados.

Palabras clave: relación amiga, τ-factorización, τ-grafo.

3 Talleres/Workshops

3.1 Métodos de investigación en un programa de formación de maestros que incluye el estudio de lecciones y el uso de tecnología de interconectividad

Omar Hernández Rodríguez, Departamento de Estudios Graduados, Universidad de Puerto Rico en Río Piedras.
Wanda Villafañe Cepeda, Departamento de Programas y Enseñanza, Universidad de Puerto Rico en Río Piedras.
Juliette Moreno Concepción, Departamento de Estudios Graduados, Universidad de Puerto Rico en Río Piedras.
Yency Choque Dextre, Departamento de Estudios Graduados, Universidad de Puerto Rico en Río Piedras.
Gloriana González, Department of Curriculum and Instruction, University of Illinois.

En este taller compartiremos los métodos de investigación que hemos utilizado durante nuestra participación en una innovación que pretende cerrar la brecha entre los cursos de métodos y las prácticas clínicas. En la investigación incorporamos el estudio de lecciones (Lesson Study), los espacios híbridos y el uso de tecnología de interconectividad; los cuales se llevaron a cabo en las experiencias de campo (pre-prácticas) de candidatos a maestros de matemáticas de nivel secundario, los cuales estaban matriculados en un curso de metodología de la enseñanza.

Después de una breve descripción del proyecto, detallaremos:

- la forma cómo hemos recopilado los datos,

- su preparación para el análisis,

- las estrategias que hemos utilizado para el análisis,

- cómo atendemos asuntos de validez y confiabilidad

- cómo informamos los hallazgos.

Las personas que participen tendrán la oportunidad de hacer prácticas con datos preparados específicamente para este taller. En específico, discutiremos algunos aspectos a considerar en la recolección y almacenamiento de la información, los retos debido a la pandemia, la protección de seres humanos participantes en investigación y los ajustes necesarios para proteger a los estudiantes que no desean participar. Posteriormente, describiremos las recomendaciones de Herbst et al. (2011) para la preparación de los datos e indicaremos la forma cómo hemos incorporado los marcos teóricos en el análisis. Consideraremos algunos aspectos relacionados a la validez y confiablidad de los análisis. Utilizaremos como ejemplos los artículos aceptados para publicación Choque, et al. (2020); González, et al. (2021) y Hernández, et al. (2020).

Finalmente, contaremos nuestra experiencia en la redacción de los informes de investigación y los procesos de someter y revisar los manuscritos para la publicación en revistas profesionales arbitradas.

Se recomienda que los participantes tengan acceso a Excel, Word y Microsoft Stream.

3.2 An invitation to string topology

Manuel Rivera, Department of Mathematics, Purdue University.

String topology is the study of the mathematical structure behind different types of interactions of curves on a geometric space. More precisely, it is the study of algebraic structures constructed by intersecting, cutting, and reconnecting families of curves on a smooth manifold. The algebraic structures studied are motivated by physical phenomena that may be observed in fluids as demonstrated in the following video: https://www.youtube.com/watch?v=PHBEMu6E3rw

The field of string topology began with algebraic constructions associated to curves on surfaces (2-dimensional manifolds) as studied by W. Goldman in 1986. For example, given two intersecting curves on a surface one may slightly deform them so that all of the intersections are transversal. Then one can concatenate curves at the intersection points to obtain new curves. Goldman described how this idea may be used to construct a Lie algebra structure on the set of linear combinations of homotopy classes of curves on a surface. In 1999, M. Chas and D. Sullivan described a rich family of operations generalizing Goldman's construction to manifolds of arbitrary dimension using techniques of algebraic topology. The totality operations may be then organized from a purely combinatorial perspective in terms of families of diagrams and graphs which in turn describe intricate algebraic structures. Chas and Sullivan's motivation was to construct new algebraic invariants that could characterize and distinguish manifold structures. In the last 20 years, string topology has been described in terms of different, but interlocking, perspectives. For example, string topology operations may be constructed rigorously using tools from homotopy theory, symplectic geometry, or homological and homotopical algebra. String topology has also interacted with other areas of mathematics such as fluid dynamics and mathematical physics. Exploring the interaction of string topology with other fields of mathematics is currently an active field of research. There are multiple open problems, questions, and projects that are accessible to mathematicians of different levels.

This introductory workshop on the subject of string topology is accessible to advanced undergraduates in mathematics who have some working knowledge of basic topology and linear algebra.

4 Charlas Concurrentes / Concurrent Talks

(In alphabetical order using the last name of the <u>speaker</u>.)

Degenerate multi-term fractional integro-differential equations and applications

Rafael Aparicio, Statistical Institute and Computerized Information Systems, University of Puerto Rico at Río Piedras.
Valentin Keyantuo, Department of Mathematics, University of Puerto Rico at Río Piedras.

We present some examples of concrete problems involving (degenerate) multi-term fractional differential equations and study their well-posedness with the method of operator-valued Fourier multipliers. The models considered appear in several applied contexts such as anomalous diffusion, electromagnetic theory, acoustics, viscoelasticity, rheology, aerodynamics, electrochemistry, cosmology, and many more.

Keywords: well-posedness, maximal regularity, operator-valued Fourier multiplier, fractional derivative

Solving the resource constrained project scheduling problem using deep learning

Roxana K. Aparicio Carrasco, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Edgar Acuña Fernandez, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The resource constrained project-scheduling problem (RCPSP) is an NP-Hard problem concerned with optimally satisfying a given set of objectives by effectively utilizing resources, subject to certain constraints.

Many researchers have tried to solve this problem with diverse methods, such as Genetic Algorithms (GA), Tabu search and other heuristic methods. On the other hand, current research shows that machine learning methods have great potential for solving the RCPSP effectively and efficiently.

In this talk we will discuss the use of Convolutional Neural Networks (CNN) to solve a simplified RCPSP problem, known as the Job Shop Scheduling Problem. In order to train the model, we first generate optimal schedules to a known benchmark problem using GA. Then, we make a conversion of the problem into multidimensional vectors of features that are used as input to the CNN. The developed CNN scheduler is then tested against the solution of the GA. Preliminary results show that the trained CNN achieves better results than other methods.

Keywords: deep learning, convolutional neural networks, scheduling, machine learning, genetic algorithms

Mean approximate controllability properties for Hilfer time-fractional differential equations

Ernes Aragonés, Department of Mathematics-Physics, University of Puerto Rico at Cayey.
Valentin Keyantuo, Department of Mathematics, University of Puerto Rico at Rio Piedras.

In this talk we discuss the mean approximate controllability of fractional partial differential equations with the so-called Hilfer type time-fractional derivative with a non-negative self-adjoint operator A_B with a compact resolvent on $L^2(\Omega)$, where $\Omega \subset \mathbb{R}^N$ ($N \geq 1$) is a bounded open set. More precisely, we show that if $1 < \mu \leq 2$, $0 \leq \nu \leq 1$ and $\Omega \subset \mathbb{R}^N$ is a bounded open set, then the system

$$\begin{cases} \mathbb{D}_t^{\mu,\nu} u(x,t) + Au(x,t) = f|_\omega(x,t) & \text{in } \Omega \times (0,T), \\ (I_t^{(1-\nu)(2-\mu)} u)(\cdot,0) = u_0, \quad (\partial_t I_t^{(1-\nu)(2-\mu)} u)(\cdot,0) = u_1 & \text{in } \Omega, \end{cases} \tag{1}$$

is mean approximately controllable for any $T > 0$, $u_0 \in V_{\frac{1}{\mu}} = D(A_B^{\frac{1}{\mu}})$, $u_1 \in L^2(\Omega)$ and any non-empty open set $\omega \subset \Omega$. The operator A_B can be the realization in $L^2(\Omega)$ of a symmetric, non-negative uniformly elliptic second order operator with Dirichlet or Robin boundary conditions, or the realization in $L^2(\Omega)$ of the fractional Laplace operator $(-\Delta)^s$ ($0 < s < 1$) with the Dirichlet exterior condition, $u = 0$ in $\mathbb{R}^N \setminus \Omega$, or the nonlocal Robin exterior condition, $\mathcal{N}^s u + \beta u = 0$ in $\mathbb{R}^N \setminus \overline{\Omega}$.

Keywords: fractional differential equations, Mittag-Leffler function, approximate controllable, mean approximately controllable

Boosting para datos infinito dimensionales

Jairo A. Ayala-Godoy, Instituto de Estadística y Sistemas Computarizados de Información, Facultad de Administración de Empresas, Universidad de Puerto Rico en Río Piedras.
Rosa E. Lillo, Departamento de Estadística e Instituto de Big Data UC3M-Santander, Universidad Carlos III de Madrid, Madrid, España.

Los algoritmos Boosting provienen del campo de aprendizaje automático en el contexto de aprendizaje supervisado. Su funcionamiento consiste en un proceso iterativo que combina clasificadores débiles (débilmente correlacionados con la clasificación correcta), con el fin de obtener un modelo predictivo robusto (mejor desempeño que cualquier clasificador débil). En la actualidad, han sido ampliamente estudiados en el campo de datos finito dimensionales, dando lugar a trabajos tanto experimentales como teóricos.

En esta charla discutiremos un nuevo enfoque de los algoritmos Boosting, ahora trabajando en el campo de datos infinito dimensionales, tema de investigación

propuesto en el trabajo de tesis doctoral realizado por el autor principal. El problema se analiza desde dos puntos de vista, (1) adaptando algunos clasificadores de datos infinito-dimensionales para incorporarlos en algoritmos Boosting existentes. (2) modificando la estructura de algunos algoritmos Boosting, extendiendo su aplicación a datos infinito dimensionales. La discusión se realiza

apoyada por estudios de simulación y se ilustra además con conjuntos de datos *Benchmark*.

Palabras clave: algoritmos boosting, clasificación supervisada, datos infinito dimensionales

A new construction of families of 3d watermarks based on column sequence

Cesar F. Bolaños Revelo, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Dorothy Bollman, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Digital watermarking is a technique used for copyright protection of easily accessible digital media, such as video and images that can be found abundantly on the internet and that requires repeated verification to detect if it has not been fraudulently manipulated. Watermarks embedded in the spatial domain directly modify pixels by replacing them by families of periodic arrays with good auto- and cross-correlation with the objective that they be undetectable by all except the owner who can unambiguously detect the watermark without requiring the original.

In this presentation we describe a new construction of families of 3D watermarks based on shifts of a Sidelnikov column sequence where the shifts are given by quadratic logarithmic maps defined on the direct product of two cyclic groups of coprime orders. The results between any 3D arrays of these families have a peak autocorrelation on the order p^4 and a non-peak auto- and peak cross-correlation maximum on order of p^2.

Keywords: autocorrelation, cross-correlation, Sidelnikov sequence, watermarks

The expected number of distinct patterns in a random permutation

Isabel Byrne, Virginia Polytechnic Institute and State University.
Verónica Borrás-Serrano, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Nathaniel Veimau, Swarthmore College.

Let π be a uniformly chosen random permutation on $[n]$. Analyzing the interaction between the numbers in two overlapping sets of k-positions, denoted π_1 and π_2, we have found the probability that the sets are order isomorphic. This probability comes from a critical lemma which proves that, in order for two sets to be order isomorphic, their overlap entries must also be order isomorphic. Once we have established the probability that π_1 is order isomorphic to π_2, we will use this to find the expected value of the number of distinct permutations in π by defining a function of the overlap. This function is based on a second lemma where we show that the number of repeated patterns is less than or equal to the number of pairs of order isomorphic patterns, which helps us to derive an upper bound on the required expectation. We will describe our significant progress in showing that the expected number of distinct permutations is $2^n(1 - o(1))$.

Boolean functions fixed under the action of the Dihedral group

José E. Calderón Gómez, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Carlos A. Molina Salazar, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Luis A. Medina, Department of Mathematics, University of Puerto Rico at Rio Piedras.

In this talk we give an explicit representation of Boolean functions that are fixed under the action of the Dihedral group. These functions are good candidate for some implementations in cryptography. We also show that the sequence exponential sums of these type of functions is a C-finite sequence and provide explicit recursion for some examples.

Keywords: exponential sums, dihedral group, C-finite sequences, balancedness

Balancing fiscal and mortality impact of COVID-19 mitigation measurements

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José Caraballo-Cueto, Graduate School of Business Administration, University of Puerto Rico at Rio Piedras.

An epidemic carries human and fiscal costs. For imported pandemics, the first-best solution is to restrict national borders to identify and isolate infected individuals. However, when that opportunity is not fully seized and there is no preventative intervention available, second-best options must be chosen. We develop a system of differential equations that simulate both the fiscal and human costs associated with different mitigation measurements. After simulating several scenarios, we conclude that herd immunity (unleashing the pandemic) is the worst policy when considering human cost. If life is disregarded, the lowest fiscal cost is found with a very strict policy (e.g., lock-down) that lowers the probability of infection by 90% for eight weeks. If fiscal expenditures are secondary, the lowest human cost is found with a relatively strict measure (e.g. physical distancing with massive testing) that lowers the probability of infection by 80%. In the case of the US, this relatively strict policy would save 910,064 lives while cost almost 10% more to taxpayers when compared to the herd-immunity case.

Keywords: COVID-19, social distancing, epidemic model, fiscal impact, physical distancing

On Bartoli-Schmidt and Kopparty-Yekhanin lemmas and its applications

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Carlos Agrinsoni, Department of Mathematics, University of Puerto Rico at Rio Piedras.
Heeralal Janwa, Department of Mathematics, University of Puerto Rico at Rio Piedras.

Absolute irreducibility property has many applications in several areas of mathematics such as algebraic geometry, coding theory, cryptography, combinatorics ans others. In this talk we discuss the classical Kopparty-Yekhanin lemma and the recent Bartoli-Schmidt lemma (or a generalization by Agrinsoni, Janwa and Delgado) and its applications to absolute irreducibility of multivariate polynomials over finite fields of characteristic two.

Keywords: irreducibility, absolute irreducibility, tangent cone, first cone

Explicit formulas for involutions of the form $x^m(x^{\frac{q-1}{2}} + a)$ with a prescribed number of fixed points over finite fields

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Ariane Masuda, Department of Mathematics, New York City College of Technology.
Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

Permutations over finite fields have many applications ranging from cryptography and combinatorics to theory of computation. For many of these applications it is important to find permutations with a small memory footprint that are easy to implement. A good option is to use permutations generated by polynomials that are their own inverse, called *involutions*. In 2017, Castro et al. gave explicit formulas for monomial involutions over $\mathbb{F}_q$ and their fixed points. The number of fixed points is important in applications in cryptography since it is related to the non-linearity of a permutation. In 2018, Zheng et al. characterized involutions of the form $x^m h(x^s)$ over $\mathbb{F}_q$. In this talk we present a refinement of their work for families of binomial involutions $x^m(x^{\frac{q-1}{2}} + a)$ over $\mathbb{F}_q$ by providing explicit formulas for m in terms of their fixed points.

Keywords: permutation polynomials, involutions, fixed points, finite fields

DNA Codes

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Verónica Borrás-Serrano, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Nathaniel Veimau, Department of Mathematics, Swarthmore College.

The computational biology field has had an enormous rise during the past decade because of all the medical advantages it has brought. DNA molecules, as a useful biological source, have also had their important role in this development through genetic codes and tests for diverse illnesses. Through algebraic coding theory, a code was designed such that all strings were distinguishable after applying mutations. Two new lemmas of binary codes allowed us to map binary code words to finite fields such that the resulting code word meets several biological and mathematical constraints.

The perceived impact of mathematics competitions on teachers and their classrooms in Puerto Rico, Switzerland and the UK

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Luis F. Cáceres-Duque, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Meike Akveld, ETH Zürich, Switzerland.
David Crawford, Leicester Grammar School, Leicester, UK.

In this talk we present the results of a small-scale, comparative study on the perceived impact that having students enter Mathematics competitions has on Mathematics teachers in Puerto Rico, Switzerland and the UK and on their classroom practice. The study surveyed a small number of Mathematics teachers in the three countries who teach in both public and private schools and in both rural and urban regions. The perceived advantages and disadvantages to students from taking part in competitions and to teachers who have students taking part in competitions are discussed and the findings compared across the three countries. The effect that Mathematics competitions have on the identification and development of the mathematical talent of students is considered together with the contribution of these activities to the academic environment of the classroom. For the teachers who did have students taking Mathematics competitions, the limitations of entry and the different methods in which teachers prepare or assist students to prepare for the competitions are compared between countries. Since the study is small in scale, no firm conclusions are drawn but suggestions are made as to where future, larger scale, studies might be carried out to see if the classroom experiences of all could be positively influenced by exposure to Mathematics competitions.

Keywords: mathematics competitions, teachers, students

Lean programming, Euclid's elements and circle packings

André Hernández Espiet, Department of Mathematics, Rutgers University.
Alex Kontorovich, Department of Mathematics, Rutgers University.

Computer assisted proofs have become more popular than ever in the last decade. Recently even some peer reviewed papers have been verified using this kind of software. In this talk, I will introduce Lean Programming, give examples of theorems that have been proved in it, and talk about some work I have done in it with regards to Euclid's elements, synthetic geometry, and circle packings.

Keywords: computer assisted proofs, Lean programming, synthetic geometry, foundations

Pronóstico de graduación de estudiantes subgraduados Universidad de Puerto Rico Recinto Mayagüez

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Jesús D. Hernández Londoño, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

Las tasas de graduación de universidades en Puerto Rico son alarmantemente bajas. En promedio, solo el 45.75% de estudiantes subgraduados obtienen su grado a 150% del tiempo. Aunque algunas universidades ofrecen talleres y consejería para estudiantes para facilitar que se gradúen, no existe una manera objetiva de predecir si una estudiante en particular completará su grado. Usamos un cohorte de 24,432 estudiantes admitidos al Recinto Univesitario de Mayaguez (RUM) entre el 1999 hasta el 2010; el cual incluye variables como programa de estudio del estudiante, puntuación en pruebas estandarizadas de entrada, nivel de educación de los padres y otras variables. Con estos datos construimos modelos para predecir si un estudiante completará su grado universitario en el RUM. Usamos 6 métodos de aprendizaje automatizado incluyendo Boosting y TabNet (una nueva arquitectura de red neuronal artificial para datos tabulares) para predecir si un estudiante subgraduado se gradúa al 150% del tiempo del RUM. Boosting y TabNet mostraron el mejor desempeño de todos los métodos al predecir si un estudiante se gradúa al 150% del tiempo del RUM. Dicha herramienta le permite a oficiales de la universidad detectar estudiantes específicos y desarrollar estrategias de intervención para aumentar sus posibilidades de graduación.

Palabras clave: aprendizaje automático, boosting, TabNet

A bayesian nonparametric model for freely-diffusing smFRET data

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Roberto Rivera Santiago, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Experiments using single molecule Förster resonance energy transfer (smFRET) techniques have greatly contributed to studies regarding conformational states of significant biological molecules. Such progress allows further examination of complex dynamics of highly flexible systems such as intrinsically disordered proteins. The goal of our work is to capture the conformational dynamics during a freely diffusing smFRET experiment. This is accomplished by representing a Hierarchical Dirichlet Process (HDP-HMM) framework in a way that distinguishes between photon-bursts (enzyme is in front of the camera) and background noise (enzyme is not in front of the camera). We present a Python code that can model surface immobilized smFRET experiments and discuss how a Markovian model with an auxiliary background state will be used to extend the Python algorithm to also capture conformational dynamics from freely diffusing smFRET experiments.

Unique properties of Cosserat elastic plates

Roman Kvasov, Department of Mathematics, University of Puerto Rico at Aguadilla.
Lev Steinberg, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

In this talk we present the unique properties of the Cosserat elastic plates according to the Cosserat Plate Theory developed by the authors. The static and dynamic properties are the direct consequence of the presence of the microstructure in the material of the plate. We discuss the numerical experiments showing that the microstructure increases plate's stiffness and dramatically changes the effect of the size of the plate on its deformation. The Cosserat Plate Theory predicts the existence of the additional Cosserat resonances of the plate free vibration, which are shown to be due to the microstructure. The numerical experiments also demonstrate the dependency of the Cosserat frequencies on the shape and the orientation of the microstructure elements (dynamic anisotropy).

La incertidumbre de una probabilidad

Alvaro Lecompte Montes, Universidad Interamericana de Puerto Rico en San German.

Se presenta una extensión de la relación de desorden o incertidumbre de Uhlman, originalmente definida para los estados de un sistema cuántico, a las medidas de probabilidad. Cuando las probabilidades son discretas, la definición es la misma de Uhlman. Cuando son probabilidades de distribución continua en o en un espacio de medida arbitrario, la relación se define comparando las funciones que dan la máxima probabilidad para un subconjunto de medida dada para cada probabilidad. Esta función se puede llamar función de certeza y es también una distribución de

probabilidad sobre los reales positivos que equivale en cuanto a certeza con la probabilidad original. Igual que en el caso cuántico, las propiedades de esta relación corresponden a lo que se espera intuitivamente sobre desorden o incertidumbre, o sus contrarios, orden y certeza, y se puede extender a álgebras no-conmutativas más generales. Las medidas de entropías usadas en física o teoría de la información, por ejemplo, dan valores más altos para las distribuciones de probabilidad más inciertas. La función de máxima certeza es interesante desde el punto de vista de certeza e información. Su valor esperado y la varianza son también medidas de incertidumbre, comparables a las entropías usuales.

El problema de pertenencia a ideales del anillo de polinomios con coeficientes enteros

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En esta charla discutimos dos algoritmos para determinar si un polinomio arbitrario con coeficientes enteros pertenece o no a un ideal en el anillo de polinomios sobre los enteros. Esto es, dados $f, f_1, \ldots, f_n \in \mathbb{Z}[x]$, el problema consiste en determinar si f pertenece o no al ideal $I := \langle f_1, \cdots, f_n \rangle$.

El primer algoritmo (Simmons (1970)) se divide en dos procedimientos. El primer procedimiento consiste en enumerar todas las n-tuplas $(g_1, \ldots, g_n) \in \mathbb{Z}[x]^n$ y calcular las combinaciones lineales $f_1 g_1 + \cdots + f_n g_n$. Este procedimiento se detiene si $f = f_1 g_1 + \cdots + f_n g_n$ para alguna de las n-tuplas; pero si f no pertenece a I, nunca se detiene. En el segundo procedimiento, primero se determina si f pertenece o no a I sobre $\mathbb{Q}$. Si $f \notin I$ sobre $\mathbb{Q}$, entonces $f \notin I$ sobre $\mathbb{Z}$, y el procedimiento se detiene; pero si $f \in I$ sobre $\mathbb{Q}$, entonces se realiza otro algoritmo efectivo. Estos algoritmos estudiados siempre paran y deciden la pertenencia.

El segundo algoritmo depende de encontrar la base mínima (Szekeres (1952)) del ideal I. Al dividir el polinomio f entre el polinomio de mayor grado en la base mínima, es casi inmediato dar respuesta al problema de pertenencia, y de ahí la importancia de esta base

Palabras clave: problema de pertenencia, dominio entero, ideales, polinomios

Un método semiparamétrico para estimar excesos de muerte durante emergencias según múltiples causas de muerte

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Determinar el número total de muertes ocasionadas en un estado de emergencia no es una tarea fácil, debido a que grandes emergencias pueden ocasionar colapso de infraestructura, o pueden ser situa-

ciones innovadoras tal que las autoridades desconocen como lidiar con ellas. Esta investigación se enfoca en construir un modelo de regresión Poisson semiparamétrico para estimar si algunas causas de muertes como accidentes por caídas y transporte, ciertas Enfermedades No Transmisibles (ENT), causas no definidas, homicidio, neumonía e influenza, sepsis, suicidio y otras causas indirectas aumentaron tras un estado de emergencia, incorporando en el modelo la variabilidad intra anual y el desplazamiento de la población.

Para ajustar las estimaciones de población durante el periodo post-emergencia, se consideran las estimaciones anuales de la población registradas mediante los datos del Censo y el movimiento neto mensual de pasajeros. Por otro lado, mediante la función spline de regresión cúbica cíclica penalizada se captura la variabilidad intra anual de mortalidad. Adicionalmente, en el modelo se implementan funciones de índices de regresión semiparamétrica para evaluar, si se presentan efectos de cambio en el comportamiento de ciertas causas de muertes en diferentes periodos post-emergencia.

La metodología propuesta es ilustrada con los datos de defunción de Puerto Rico entre los años 2014 a 2018 para estimar si las causas mencionadas, presentaron un exceso de mortalidad tras el paso del Huracán María.

Palabras clave: modelo de regresión Poisson semiparamétrico, variabilidad intra anual, desplazamiento poblacional, función spline de regresión cúbica cíclica penalizada, enfermedades no transmisibles (ENT)

Improving the minimum distance bounds of trace Goppa codes

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Natalie Dodson, Middlebury College.
Ryan Lynch, University of Notre Dame.
Eric J. Pabón Cancel, University of Puerto Rico at Mayagüez.

Binary Goppa codes are one of the most widely studied classes of linear codes. They have a good decoder available and they resist cryptographic attacks. Several of the best known linear codes are Binary Goppa codes. In this talk we shall present an improvement on the minimum distance of Trace Goppa Codes, a class of linear codes where the Goppa polynomial is $\mathrm{Tr}_{\mathbf{F}_{q^m}/\mathbf{F}_q}$. P. Veron derived improvements on their dimension and minimum distance. We improve the previous distance bound from $2q^{m-1} + 1$ to $2(q^{m-1} + q^{m-2} + \cdots + q) + 1$. When $m = 3$, we make a further improvement to $2q^2 + 2q + 8$. These are significant improvements for trace Goppa codes from non-quadratic extensions.

Creación de un bachillerato en ciencia de datos

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La Junta de Gobierno de la Universidad de Puerto Rico aprobó la propuesta del Departamento de Matemáticas de la UPR en Humacao para la creación de un nuevo Bachillerato en Ciencia de Datos (BCD). Para esta propuesta se compararon los contenidos curriculares de varios programas en Ciencia de Datos de otras universidades. Dado que la disciplina es relativamente nueva se observó considerable diversidad en los contenidos curriculares y orientación de estos programas. Nuestra propuesta optó por considerar los componentes de ciencia de cómputos, matemáticas y estadística. El contenido curricular del programa propuesto se puede considerar como una reorientación y actualización del contenido curricular del Programa de Bachillerato en Matemáticas Computacionales que se ofrece actualmente.

El BCD es significativamente distinto a otros programas afines de la UPR y universidades privadas. Lo cual, a la fecha, lo hace único en Puerto Rico. Su contenido curricular se divide en tres categorías: Educación general, concentración y electivas. Dentro de la categoría de concentración, los cursos de especialidad son planteados bajo los enfoques de aprendizaje cooperativo y aprendizaje basado en proyectos. Esta categoría también contempla opciones para experiencias de integración bajo la forma de proyectos de investigación o proyectos de diseño e implementación de soluciones usando ciencia de datos.

En esta charla se describirán los aspectos de la propuesta antes mencionados. Así como los planes y retos de implantación luego de la aprobación de la misma.

Palabras clave: ciencia de datos, desarrollo curricular

The brachistochrone problem over surfaces

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We consider the problem of finding curves of minimum time of descent, joining two given points over a given frictionless surface and under the influence of gravity. We discuss the existence and minimality of extremals for the corresponding time functional, and find explicit solutions in certain special cases, including a closed form solution for the problem on an inclined plane. A discussion of numerical methods for computing these minimizers is given with several numerical examples for which explicit solutions are not known. This paper is mostly expository. Our main contribution is to put together on a single reference different results concerning the existence and minimality of extremals, explicit solutions in special cases, and a discussion of numerical methods for computing the desired minimizers.

Keywords: brachistochrone, parameterized surface, calculus of variations, minimality of extremals

Cavitation of a spherical body under mechanical and self gravitational forces

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In this paper we look for minimizers of the energy functional for isotropic compressible elasticity taking into consideration the effect of a gravitational field induced by the body itself. We consider two type of problems: the displacement problem in which the outer boundary of the body is subjected to a Dirichlet type boundary condition, and the one with zero traction on the boundary but with an internal pressure function. For a spherically symmetric body occupying the unit ball $\mathcal{B} \in \mathbb{R}^3$, the minimization is done within the class of radially symmetric deformations. We give conditions for the existence of such minimizers, for satisfaction of the Euler–Lagrange equations, and show that for large displacements or large internal pressures, the minimizer must develop a cavity at the centre. A numerical scheme for approximating these minimizers is given together with some simulations that show the dependence of the cavity radius and minimum energy on the displacement, internal pressure, and mass density of the body. We give a numerical example with values characteristic of the composition of the planet Mercury.

Keywords: nonlinear elasticity, cavitation, self-gravity, internal pressure

Exploring computer science for Puerto Rico: results and experiences from a four-year journey

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A group of educators from the Department of Computer Science teamed with the College of Education of the UPR-RP to address the need for integrating Computer Science Education in Puerto Rico's high schools. From 2017 to 2021, the National Science Foundation (NSF) funded the project Exploring Computer Science for Puerto Rico (ECS4PR). ECS4PR is facilitating the implementation of the Exploring Computer Science (ECS) curriculum in Spanish, Explorando el mundo de la computación, through the Professional Development (PD) of school teachers. Three key aspects in this process are: First, the establishment of a researcher-practitioner partnership (RPP). Second, the cultural and linguistic adaptation of it, beyond a simple translation of the ECS curriculum. Third, the scaling-up of the implementation of the curriculum in the Puerto Rico Department of Education. In this work, a) we share the results and experiences regarding the three-year implementation of teacher cohorts, describe the development of the RPP and the ECS PD workshops, including the adaptation of the latter to an online version with experienced teachers as co-trainers; b) present our approach for the cultural and linguistic adaptation of the ECS curriculum, including its adaptation to a newer version (V9.0); c) describe the scaling-up of the ECS PD, and its future implementation, to a broader community of educators with the commitment of the Puerto Rico Department of Education. Finally, we share our data from: (1) repeated assessments on the development of the RPP; (2) pre/post-survey assessments of the ECS PD for teacher participants; (3) students' assessment developed by SRI International; and (4) other instruments that assess

teachers' perceptions about the translated ECS curriculum, and their efforts to implement it. At present, we have a stable RPP community that has evolved to address teachers' needs through peer-coaching and teacher-led workshops. We are presently in the process of planning the submission of a new proposal to expand, research, and enhance the implementation of the ECS curriculum in Spanish in the Puerto Rico Department of Education.

Keywords: exploring computer science, ECS, professional development, computer science education, researcher-practitioner partnership

Debilidades conceptuales encontradas en los estudiantes de Precálculo I de la UPRM

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En este trabajo se presentarán los hallazgos generales encontrados en un grupo de 736 estudiantes de nuevo ingreso al curso de Precálculo I en la Universidad de Puerto Rico, Recinto de Mayagüez (UPRM), en cuanto a lo que se ha llamado debilidades conceptuales y operacionales, es decir, lo que el estudiante previamente debería saber antes de entrar en contacto con la temática a desarrollarse en el curso de Precálculo I.

Se presenta un resumen general sobre un sondeo semanal realizado con la colaboración de 13 docentes o instructores del área de matemáticas quienes sostuvieron clases con estos estudiantes durante el primer semestre del 2021 – 2022. Estos instructores semanalmente reportaron todos aquellos conceptos previos que, según el criterio de cada uno de ellos, sus estudiantes deberían saber y no lo saben, antes de entrar en contacto con los conceptos del curso. Se presentará el análisis de estos resultados.

Este trabajo constituye la primera parte del estudio bajo la modalidad de "Corequisite Support" en la UPRM en el curso de Precálculo I. En el presente semestre se está llevando a cabo la segunda parte que consiste en la implementación de un plan de refuerzo académico basado en los hallazgos de la primera parte.

Palabras clave: Precálculo I, debilidades conceptuales y operacionales, conceptos previos, sondeo

Improving bounds of hermitian-lifted codes with their automorphism group

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Locally Recoverable Codes (LRCs) are linear codes where the value of any single position may be recovered from a small subset of positions. There are several combinatorial and algebraic constructions of LRCs. For example, Lifted Reed-Solomon codes are LRCs whose recovery sets are point sets from the lines of the affine plane. In this work, we improve the bound of the dimension of the Hermitian-Lifted Codes. With the automorphism of the Hermitian curve we simplify the code's basis and easily check the LRC conditions holds on each line.

Keywords: locally recoverable codes, monomials, hermitian curve

Monitoreo de calidad de agua de ríos en Puerto Rico usando modelos aditivos mixtos con P-splines

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Los modelos lineales mixtos son una generalización de los modelos lineales y son ampliamente utilizados en diferentes ramas de la ciencia, pues permiten modelar estadísticamente la posible correlación en los datos. Sin embargo, en muchos casos los datos no están relacionados de forma lineal y a pesar de que se pueden implementar los modelos de regresión no-lineal clásicos, en donde se impone una función a priori que relacione la variable respuesta con las variables predictoras, pueden existir casos en donde la estructura de los datos es más compleja y haga difícil encontrar una función que estime la relación existente de forma correcta. Una alternativa a situaciones como esta es la utilización de modelos aditivos mixtos, en donde la función suave que relaciona el valor medio de la variable respuesta y con variables predictoras no es especificada a priori, sino que es construida mediante splines con penalizaciones (P-splines).

En este trabajo analizamos 57 ríos en Puerto Rico para estudiar las tendencias a lo largo del tiempo de los parámetros de calidad del agua, como el pH, la temperatura del agua, la concentración de oxígeno disuelto, la concentración de fósforo, etc., bajo modelos aditivos mixtos. Los datos se

obtuvieron de los registros del Servicio Geológico de EE. UU. y varios estudios adicionales realizados en Puerto Rico en los últimos 15 años. Los datos disponibles abarcan desde 1958 hasta 2019, aunque existen brechas significativas en varios años. Los modelos aditivos mixtos se ajustaron utilizando la librería mgcv (R v. 4.0.5). La naturaleza longitudinal de estos datos se consideró utilizando efectos aleatorios apropiados. También incorporamos covariables significativas para estudiar las diferencias entre regiones y el impacto de las plantas de tratamiento de agua. Se ajustaron funciones suaves para estudiar estos efectos a lo largo del tiempo utilizando P-splines. Las interacciones estudiadas explicaron las tendencias observadas y arrojaron predicciones específicas de la condición, que se pudieran comparar mediante la construcción de matrices de contraste apropiadas. Con este enfoque también pudimos evaluar cambios estadísticamente significativos en las tendencias debido a intervenciones específicas en el tiempo y predecir las tendencias actuales en los parámetros de calidad del agua.

Palabras clave: splines, modelos lineales mixtos, modelos aditivos mixtos

The orbit structure of the Grassmannian and a simplified decoder for $C(2, m)$

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In this presentation, we decode Grassmann codes, linear codes associated to Grassmannian of lines. We look at the orbit structure of Grassmannian arising from the natural action of multiplicative group of certain finite field extension. We project the corresponding Grassmann code onto these orbits to obtain a few subcodes of certain Reed-Solomon code. We prove that most of the projected codes contain an information set of the Grassmann code. Using Peterson's decoding algorithm for the projected subcodes, we prove that one can correct up to $\lfloor d - 1/2 \rfloor$ errors for Grassmann code, where d is the minimum distance of the Grassmann code $C(2, m)$.

Keywords: Grassmann codes, Reed–Solomon decoder, decoding

Diophantine equations with binomials coefficients and perturbations of symmetric Boolean functions

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Luis A. Medina, Department of Mathematics, University of Puerto Rico at Río Piedras.

In this talk we study perturbations of symmetric Boolean functions and establish a connection between exponential sums of these functions and Diophantine equations of the form

$$\sum_{\ell=0}^{n} \binom{n}{\ell} x_\ell = 0,$$

where x_ℓ belongs to some bounded subset Γ of $\mathbb{Z}$.

Matemáticas y olimpiadas durante la pandemia, retos y oportunidades

Arturo Portnoy, Departamento de Ciencias Matemáticas, Universidad de Puerto Rico en Mayagüez.

Se compartirán algunas experiencias educativas vividas durante la pandemia, como talleres de olimpíadas para estudiantes de nivel elemental, y adaptación de diferentes cursos de matemáticas. Se discutirá la tecnología utilizada, cambios y ajustes en la filosofía de la enseñanza, así como retos y dificultades, incluyendo el plagio y la evaluación. Finalmente compartiré opiniones sobre las oportunidades que brinda la educación a distancia, y la necesidad de reexaminar el rol del maestro y la visión tradicional de la educación a la luz de todas estas tecnologías que se han hecho cotidianas de forma acelerada.

Ciencia de datos en un curso introductorio de programación

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Presentamos nuestras experiencias enseñando un curso introductorio de programación, para estudiantes de primer año de universidad, el cual se revisó para exponer a los estudiantes a temas y herramientas de la ciencia de datos. Entre las experiencias se incluyen: el proceso de limpieza y depuración de datos, manejo de conjuntos de datos "grandes", visualización de datos, manejo y programación con APIs (Application Programming Interface), rudimentos de los bancos de datos, y diseño de aplicaciones para la web. Discutimos algunas de las dificultades y oportunidades que surgen al combinar el proceso de enseñanza introductoria con experiencias y metodologías que usualmente se discuten en cursos más avanzados. Este curso, es parte de la secuencia curricular del nuevo Bachillerato en Ciencia de Datos en la Universidad de Puerto Rico en Humacao.

Palabras clave: ciencia de datos, desarrollo curricular

The Korteweg-deVries equation on the line

Brian Reyes Vélez, Department of Mathematics, University of Notre Dame.

We study the Cauchy problem for a Korteweg-deVries equation with initial data in Sobolev Spaces and prove local well-posedness for H^s, $s > -\frac{3}{4}$. This is done by first proving key bilinear estimates in appropiate Bourgain spaces.

We consider the initial value problem (ivp) for the Korteweg-deVries equation

$$\partial_t u + \partial_x^3 u + \frac{1}{2}\partial_x(u^2) = 0, \tag{2a}$$

$$u(x,0) = \varphi(x) \in H^s(\mathbb{R}), \quad t \in \mathbb{R}, \quad x \in \mathbb{R}, \tag{2b}$$

and we will present the proof of its local well-posedness in Sobolev spaces $H^s(\mathbb{R})$ following the work on KdVm by Alex Himonas & Fangchi Yan & Renata Figueira (2020) for the more general m-th order dispersion KdV (KdVm) which in turn is a continued work of Kenig, Ponce and Vega from (1996).

In 1993 Bourgain introduced the spaces $X^{s,b}$ and proved local well-posedness for $s \geq 0$. Then in 1996 this result was improved for $s > -\frac{3}{4}$ by Kenig, Ponce and Vega. Later in 2003, the local well-posedness result of KdV was extended to **global** well-posedness for $s > -\frac{3}{4}$ by Colliander, Keel, Staffilani, Takaoka and Tao. Then, it was shown by Christ, Colliander and Tao that KdV is not well-possed for $s < -\frac{3}{4}$ since the KdV solution map fails to be uniformly continuous in H^s. Finally for $s = -\frac{3}{4}$ Guo proved **global** well-posedness of KdV in H^s.

Affine hermitian Grassmann codes

Fernando L. Piñero, Department of Mathematics, University of Puerto Rico at Ponce.
Doel Rivera Laboy, Pontifical Catholic University of Puerto Rico.

In this talk we introduce a new class of linear codes, called affine Hermitian Grassmann codes. These codes are the linear codes resulting from an affine part of the projection of the polar Hermitian Grassmann codes. They combine polar Hermitian Grassmann codes and affine Grassmann codes. The Grassmannian is an important object in Algebraic Geometry. Many of its properties are studied via linear codes from their embeddings. We have determined the parameters of these codes, their dual codes and characterize their minimum distance code words.

Data science for a critical society: A curriculum project

Bradly Rivera Muñiz, Departamento de Estudios Graduados en Educación, Universidad de Puerto Rico en Rio Piedras.

Individuals who don't develop their ability to become aware of their social situation tend to see this as a cause of their own failures or are convinced that their current social situation is a given or mere casualty. Studying one's own social situation through objective means, such as through data science, can help internalize that in most cases the argument of mere casualty or failure does not hold true. Data science is a relatively emerging field that takes knowledge from statistics, data mining, databases and distributed systems and combines it in a single multifaceted domain of knowledge. These fields are of mathematical, scientific, and technology-based nature, due to the background knowledge required to understand the core concepts, the application that it can be extended to, and the dependency of technology in the implementation of procedures and algorithms. Understanding the world through data can bring objectivity to the process of interpreting the situations that are currently happening in one's community, country, or globally. This is one main reasons for the conception of Data Science for a Critical Society. Due to its practicality and extensive application power, necessity, as well as some other reasons, being a data scientists have been recognized as one of the most attractive careers in recent years. However, one must be aware of how data visualizations, which are inherently a by-product of data science, are produced narratives with assumed objectivity that justify their consumption. In essence, data can be used to objectify the interpretation of reality, but it can also be used with misleading intentions, to deceive the consumer of the data itself. In our current global information-based society data has become an asset. Some authors have been explicit about how everything we do in our daily life has the potential to become a data point, that can be used for a variety of purposes. This urgently calls for the need to be critical in understanding how data visualizations are created, what they can represent and how data, in general, is analyzed and interpreted. Projects in which curriculum for the K-12 level have been developed or are advocated for have become more common in the past years, others have focused more on the college/university level, while others have argued that modern high school mathematics should not be the traditional courses, but data science. However, many to no efforts have been dedicated to the elementary school data science education. In this talk we will present a curricular project with which we aim to cover that necessity and expose elementary school students, particularly those in the upper elementary, to data science as a domain of knowledge that is interconnected with fields such as mathematics, science, and social sciences, with the goals of providing a pathway to a more informed/objective and justice-oriented society through the development of critical citizens that can interpret the world objectively through data.

Keywords: data science, critical mathematics, elementary education, objectivity

Computing the transient, for a family of non linear discrete dynamical systems

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An open problem in the theory of discrete dynamical systems is to link the structure of a system with its dynamics. This paper contains such a link for a family of nonlinear systems over the field with two elements. For a family of systems that can be described by monomials (including Boolean AND systems), one can obtain information about the transient of the system from the structure of the monomials. In particular, this work contains a formula for the transient of monomial system that only have fixed points as limit cycles. This condition depends on the cycle structure of the dependency graph of the system.

Keywords: transient, dependency graph, state spaces

Forward-Backward Stochastic Differential Equations with Conditional Mean-Field and Regime Switching and Stochastic Differential Games

Esteban Rolón Gutiérrez, Department of Mathematics, University of at Río Piedras.
Son Luu Nguyen, Department of Mathematics, University of Puerto Rico at Rio Piedras.
George Yin, Department of Mathematics, University of Connecticut.

In this talk, we first focus on forward-backward stochastic differential equations with regime-switching. Then we focus on forward-backward stochastic differential equations with mean-field interactions and regime-switching. We obtain conditions for unique solvability of such equations without assuming the no-degenerate condition for the forward equation. The results are then applied to investigate the problem of existence of open-loop Nash equilibrium points for nonzero sum linear-quadratic stochastic differential games with random coefficients.

Keywords: switching diffusion, mean-field interaction, forward-backward stochastic differential equations, stochastic differential games

Permutation binomials of the form $x^r(x^{q-1} + a)$ over $\mathbb{F}_{q^e}$

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Ivelisse Rubio, Department of Computer Science, University of Puerto Rico at Río Piedras.

We present several existence and nonexistence results for permutation binomials of the form $x^r(x^{q-1} + a)$, where $e \geq 2$ and $a \in \mathbb{F}_{q^e}^*$. In particular, we give a formula for constructing such permutation binomials via compositions of linearized binomials and monomials. We also obtain a complete characterization of $x^r(x^{q-1}+a)$ over $\mathbb{F}_{q^2}$, $\mathbb{F}_{q^3}$, $\mathbb{F}_{q^4}$, $\mathbb{F}_{p^5}$, and $\mathbb{F}_{p^6}$, where p is an odd prime.

Preferential and k-zone parking functions

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Christopher Soto, Department of Mathematics, Queens College of the City University of New York and Department of Computer Science, Columbia University.
Pamela Vargas, Department of Mathematics and Statistics, Smith College.
Pamela E. Harris, Department of Mathematics and Statistics, Williams College.

Parking functions are vectors that describe the parking preferences of n cars that enter a one-way street containing n parking spots numbered 1 through n. A list of each car's preferences is also compiled into vectors in which we denote as $(a_1, \ldots, a_n)$, such that a_i is the parking preference for car i. The classical parking rule allows cars to enter the street one at a time going to their preferred parking spot and parking, if that space is unoccupied. If it is occupied, they then proceed down the one-way street and park in the first available parking spot. If all cars can park, we say the vector $(a_1, \ldots, a_n)$ is a parking function.

In our research, we introduce new variants of parking function rules with backward movement called k-Zone, preferential, and inverse preferential functions. We study the relationship between k-Zone parking functions and k-Naples parking functions and count the number of parking functions under these new parking rules which allow cars that find their preferred spot occupied to back up a certain parameter. One of our main results establishes that the set of non-increasing preference vectors are k-Naples if and only if they are k-Zone. For one of our findings we provide a table of values enumerating these new combinatorial objects in which we discover a unique relationship to the order of the alternating group A_{n+1}, number of Hamiltonian cycles on the complete graph, K_n, and the number of necklaces with n distinct beads for $n!$ bead permutations.

Implantación de un curso de ciencia de datos a nivel de bachillerato

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El Departamento de Matemáticas de la UPR en Humacao incluyó un curso de Introducción a la Ciencia de Datos (ICD) como requisito en su nuevo bachillerato en esa disciplina. Por ser una disciplina novel no existe consenso ni estándares de organizaciones profesionales académicas sobre el contenido de un curso como este. Muestra de ello es la ausencia de una acreditación específica para programas en ciencia de datos por parte de ABET. No todos los programas de bachillerato de Ciencia de Datos examinados (60%) tienen un curso similar en su currículo. Temas comunes en los cursos ICD en programas de bachillerato existentes incluyen el acceso y manipulación de cantidades grandes de datos, preparado para su análisis (data wrangling), representación gráfica y algoritmos básicos de clasificación automatizada. Diferencias notables pueden observarse en cuanto a la complejidad de las herramientas estadísticas y matemáticas usadas, la programación de software o el uso de programado existente y la amplitud de los temas.

En Humacao optamos por un curso cuyo objetivo es dar a la estudiante una visión panorámica de la Ciencia de Datos como disciplina en un formato similar al de los cursos generales que suelen requerir los programas en ciencias naturales. Estructuramos el curso basado en el flujo de trabajo de la Ciencia de Datos (procesamiento, depurado, análisis exploratorio, modelado matemático, producto informático y visualización).

En esta charla describiremos el proceso de determinación de necesidad, implantación del curso ICD y la primera experiencia en su ofrecimiento. Derivaremos conclusiones en cuanto a accesibilidad a estudiantes, acceso a datos y dificultades técnicas.

Palabras clave: ciencia de datos, desarrollo curricular

No three in a θ

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Dashleen González-Valentín, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
Ryan Lynch, University of Notre Dame.
Lani Southern, Willamette University.

The no-three-in-a-line problem asks for the maximum number of points that can be placed on an $n \times n$ grid such that no three of them lie in a line. It has remained unsolved for over 100 years, even though it has an easily proven upper bound of $2n$. Inspired by this problem, we propose an extension similar to the one studied by Gossell and Johnson: How many points can be chosen on an $n \times n$ grid such that no three of them form an angle of θ? We classify the angles that

yield interesting problems and focus on angles that appear in surprising configurations on the grid. We prove upper and lower bounds for specific angles and discuss the geometric properties of the grid.

Polar hermitian Grassmann codes

Sarah Gregory, University of Richmond.
Doel Rivera Laboy, Pontifical Catholic University of Puerto Rico.
<u>*Lani Southern*</u>, Willamette University.

The Grassmannian, $\mathcal{G}_{\ell,m}$, is the collection of all ℓ dimensional subspaces of a vector space of length m. It is one of the most widely studied objects in Algebraic Geometry. It has several algebraic, geometric and combinatorial properties and may be studied in several ways. We use a class of linear codes, known as Grassmann codes, to study the Grassmannian. The polar Hermitian Grassmannian is a subvariety of the Grassmannian consisting of all spaces isotropic under a sesquilinear form. The minimum distance of the corresponding linear code is related to the intersection of the Grassmannian (or its subvariety) with a hyperplane. Cardinali and Giuzzi determined the minimum distance for the case where $\ell = 2$. For the case where $\ell = 3$ and $m = 6$ we employ an elementary counting argument to prove a lower bound on the minimum distance for the polar Hermitian Grassmann codes.

Proofs for two conjectures on multidimensional periodic Costas arrays

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In a 1996 article, Solomon W. Golomb and Oscar Moreno conjectured that a Costas array is circular if and only if it is exponential Welch. Muratović-Ribić et al. recently established this conjecture affirmatively. In 2013, J. Ortiz Ubarri et al. defined multidimensional periodic Costas arrays (MPCA), a higher-dimensional analogue of circular Costas arrays. They showed that the generalized Welch construction yields MPCAs and provided four conjectures regarding algebraic generators, the number, and the size of MPCAs. By connecting MPCAs with linearized permutation polynomials, we prove one of the conjectures on the number of MPCAs. On the other hand, using direct product difference sets and their connections to finite projective geometry, we establish another of the conjectures of Ortiz Ubarri et al., the one about the size of MPCAs. These results provide evidence for a higher-dimensional analogue of the result conjectured by Golomb and Moreno.

Un modelo markoviano factorial para capturar comportamiento dinámico en experimentos usando microscopios de electrones (TEM)

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Las propiedades presentes en los nanomateriales son altamente dependientes de su tamaño, forma y composición atómica, lo que genera que posean características ópticas, electrónicas y magnéticas, diferentes y no bien conocidas.

Predecir o cuantificar con cierto grado de confianza la dinámica de la configuración atómica de las nanopartículas durante procesos de difusión superficial en experimentos in situ, es de gran interés porque nos permite conocer de antemano los cambios en su estructura atómica con la suficiente certeza para hacer un uso eficiente de sus propiedades y maximizar sus beneficios.

Proponemos utilizar un Hidden Markov Model factorial para caracterizar la dinámica de una nanopartícula de óxido de cerio cuando esta es sometida a un experimento in situ y capturada a través de imágenes TEM. Cada columna atómica en la nanopartícula es considerada una serie de tiempo y nuestro modelo se centra en estimar el número de átomos en cada columna de la nanopartícula; los estados de las cadenas Markovianas.

Multi-marginal optimal transport: uniqueness and graph theory

Adolfo Vargas-Jiménez, University of Alberta.
Brendan Pass, University of Alberta.

Multi-marginal optimal transport, a natural extension of the well-known classical optimal transport problem, is the problem of correlating given probability measures as efficiently as possible relative to a given cost function. Although a variety of applications have arisen over the past eleven years and uniqueness is well understood in the classical problem, the structure of solutions for the multi-marginal case has been much more difficult to address, mainly due to the strong dependence on the cost function. In this talk, I will briefly introduce the problem, summarize the known results for uniqueness in the multi-marginal case, and connect them with some past joint works with my supervisor Brendan Pass based on costs associated with wide classes of graphs.

Key words: multi-marginal optimal transport, Kantorovich formulation, Monge formulation, Monge solution

The generalized anisotropic dynamical Wentzell heat equation with nonstandard growth conditions

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Alejandro Vélez-Santiago, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded Lipschitz domain with the anisotropic extension property, for $N \geq 3$. We consider a new class of generalized anisotropic heat-type boundary value problems involving the anisotropic $\overrightarrow{p}(\cdot)$-Laplace operator $\Delta_{\overrightarrow{p}(\cdot)}$, with (pure) dynamical anisotropic Wentzell boundary conditions

$$u_t + \sum_{i=1}^N |\partial_{x_i} u|^{p_i(\cdot)-2} \partial_{x_i} u\, \nu_i - \Delta_{\overrightarrow{q}(\cdot),\Gamma} u + \beta |u|^{q_M(\cdot)-2} u \ni 0.$$

Here $\overrightarrow{p}(\cdot)$ and $\overrightarrow{q}(\cdot)$ are Lipschitz continuous vector fields, which can be unrelated between each other, and $\Delta_{\overrightarrow{q}(\cdot),\Gamma}$ represents the anisotropic $\overrightarrow{q}(\cdot)$-Laplace-Beltrami operator acting on $\Gamma := \partial\Omega$. We first prove that the operator $\Delta_{\overrightarrow{p}(\cdot)}$ with the above boundary conditions generates a nonlinear order-preserving submarkovian C_0-semigroup $\{T_\sigma(t)\}$ over $\mathbb{X}^{r(\cdot)}(\overline{\Omega}) := L^{r(\cdot)}(\Omega) \times L^{r(\cdot)}(\Gamma)$ for all measurable functions $r(\cdot)$ on $\overline{\Omega}$ with $1 \leq r^- \leq^+ < \infty$. Consequently, the corresponding anisotropic dynamical Wentzell problem is well-posed over $\mathbb{X}^{r(\cdot)}(\overline{\Omega})$. Furthermore, we show that the nonlinear C_0-semigroup $\{T_\sigma(t)\}$ enjoys a Hölder-type ultracontractivity property. In particular, we obtain the global boundedness of the mild solutions to the parabolic problem.

Keywords: anisotropic heat equation, anisotropic problems with variable exponents, dynamical Wentzell boundary conditions, nonlinear C_0-semigroups, ultracontractivity properties

Soft matroid: a new independence

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Carlos Araujo, Departamento de Matemáticas, Universidad del Atlántico, Puerto Colombia, Atlántico, Colombia.

In this talk, we discuss a new notion of independence, called *parametric independence*, that allows us to obtain a connection between matroids, soft sets and graphs. With this idea, we introduce the *soft matroids*, and present some properties of their basic elements. In addition, we show some graphic representations and descriptions of these matroids associated with the soft sets.

Keywords: soft set, soft matroid, parametric independence, soft points, attractors, repulsors, graph

5   Afiches / Posters

(In alphabetical order using the last name of the <u>presenter</u>.)

Modeling chemotaxis of coffee berry borers on a branch

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Mariano Marcano, Department of Computer Science, University of Puerto Rico at Rio Piedras.

Hypothenemus Hampei (Coleoptera), commonly known as Coffee Berry Borer (CBB), is a small brown beetle found in coffee fields around the globe. It is often referred to as the most important coffee pest due to the fact that it attacks the coffee berry, directly affecting harvest yields. Due to its small size and the fact that it spends most of its life cycle within the coffee berry, its behavior remained hard to study for decades. It is now known that CBBs tend to prefer ripe and dry berries and tend to avoid berries that are already infested. They distinguish the age and state of infestation of coffee berries by the different volatile compounds released by coffee berries during each stage of maturity and when infested. We seek to construct a dynamical mathematical model that captures this behavior. We begin by considering the distribution of two variables representing the concentration of CBB and a coffee chemical attractant along the length of a branch. These variables will change over time through a system of coupled partial differential equations modeling diffusion, chemotaxis, and reproduction, known as the Keller-Segel Model. We solve this model numerically through a strang-splitting method, implementing different numerical schemes for each subproblem. We consider two bunches of coffee berries modeled as two Gaussian curves with centers set 10cm apart. We then explore the dynamics that arise for different initial conditions of the CBB distribution and how these dynamics change with varying values of the constant modulating the chemotactical response.

Keywords: partial differential equations, nonlinear dynamics, numerical methods, chemotaxis, coffee agriculture

Effects of knowledge graph structural properties on their predictive performance

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Knowledge graphs (KGs), comprehensive data structures where the relations between different entities of a shared environment are represented through labeled nodes and edges, are an increasingly applied resource for organizing and extrapolating relational data. Their intuitiveness and versatility, from both a feature-based and a structural perspective, have signified a recent boom in the research of their applicability in a variety of domains, ranging from drug discovery to social media modeling. Across these fields, academic and industrial efforts have resulted in the devel-

opment of extensive literature and software tools, particularly in the form of automated learning models designed to tackle statistical learning tasks like link prediction –discovering new relations between pairs of elements in a network– or entity classification –identifying the correct label and corresponding properties of a newly introduced element. This, however, has not been reflected in an equivalent amount of published research regarding how a graph's original structural properties affect the predictive performance of these models, a question for which there seems to only be a limited amount of publicly available resources.

The aim of this investigation is to begin an exploration of this relation and extend the contributions of the few researchers which have approached it, as well as to spotlight the potential of developing tools designed specifically to identify these morphological factors and manipulate them to improve model performance over problematic subgraphs. For this purpose, we firstly identify some of these properties, namely the sparsity and probabilistic distributions of a KG's entity and relationship categories. Subsequently, we use gMark, a KG generator that allows for basic adjustment of entity proportions and distributions, to synthesize multiple iterations of knowledge graphs with shared categories, distinguished primarily by these structural characteristics. Finally, we apply a variety of graph neural networks (GNNs) over our iterations, to empirically test the impact and relevance of said characteristics over the models' performance.

Keywords: knowledge graphs, graph neural networks, link prediction, entity classification

Some Results of k-almost $\tau_{(n)}$-primes

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Reyes M. Ortiz Albino, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.

The theory of $\tau_{(n)}$-factorizations and $\tau_{(n)}$-products were first introduced and defined by Anderson and Frazier in 2006. As an example of the theory of generalized factorizations they used the relation $\tau_{(n)}$, which is the equivalence relation modulo n on the non zero, non invertible integers as a restriction on the multiplicative operation. That is, a $\tau_{(n)}$-product is the product of elements that are related under $\tau_{(n)}$. A year later, Hamon characterized the form of the integers known as the $\tau_{(n)}$-primes. Last year, Ortiz and Cruzado introduced the notion of the semi $\tau_{(n)}$-primes and provided a characterization of their form. We say an integer is a semi $\tau_{(n)}$-prime when it is the result of the product of exactly two $\tau_{(n)}$-primes. This ongoing work further studies such elements and extends the notion, as a generalization, to the k-almost $\tau_{(n)}$-primes; an integer is called k-almost $\tau_{(n)}$-prime if it is the product of k $\tau_{(n)}$-primes. We provide some examples and properties of them. To study the k-almost $\tau_{(n)}$-primes for any n and k, we review the $\tau_{(n)}$-primes and some of their characteristics.

Keywords: primes, factorizations, semiprimes, k-almost primes

Actions and Factorizations

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In 2006, Anderson and Frazier defined the notion of generalized factorizations using a restriction of the multiplicative operation with respect to a symmetric relation τ on the non-zero and non-unit elements of an integral domain. This notion can be interpreted as allowing only the elements that are related with respect to τ to multiply. This notion has been studied since 2003 through 2014 by Anderson's PhD students, but very few results have been published since. Relations are important as they provide information and new ideas to do research, and their study will look for several properties of relations. Anderson and Frazier defined three relations not seen before. However, the nature of the properties of relations need further study. The proposed research project hopes to define a type of action of the domain D on a relation τ, coordinate-wise. We expect to expand our toolset to characterize properties of relations and the potential implications they might have in the theory of generalized factorizations..

The study of $\tau_{(n)}$-primes

Eric J. Pabón Cancel, Department of Mathematical Sciences, University of Puerto Rico at Mayagüez.
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The theory of $\tau_{(n)}$-factorizations was first defined in 2006 by Anderson and Frazier; since 2007, Hamon, Ortiz, Juett, among others have developed this theory. Many results have been obtained in terms of algebraic structure classification, but very little about some types of elements that have arisen. For example, is the concept of $\tau_{(n)}$-prime elements, which are analogue to the primes in the usual product. Unfortunately the only properties we know is a characterization of their form (that is, $p_0^{\epsilon_0} \cdot p_1^{\epsilon_1} \cdots p_k^{\epsilon_k}$ is a $\tau_{(n)}$-prime where $n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_k^{n_k}$, each n_i is a non-negative integer, $\epsilon_i \in \{0, 1\}$ and the p_i's are non-associated primes). And the fact that each $\tau_{(n)}$-factorization into $\tau_{(n)}$-primes is unique, if it exists.

In our recent work, we focused on studying the distribution of $\tau_{(2)}$-primes and $\tau_{(3)}$-primes. This gave us an idea on the structure of $\tau_{(n)}$-primes distribution for other values of n. We noticed that if k is a $\tau_{(2)}$-prime, then each equivalence class modulo k contains at least one $\tau_{(2)}$-prime. This property seems to hold when k is a $\tau_{(3)}$-prime and $\tau_{(6)}$-prime as well. We suspect this also occurs in a more general form. This is analogue to the idea that the prime numbers are distributed among all equivalence classes modulo a prime. Therefore, we have studied the equivalence classes modulo

k that contain $\tau_{(n)}$-primes. From this, possible results regarding the Euler totient function may arise in the theory of $\tau_{(n)}$-products. We show examples and ongoing results.

Keywords: primes, prime distribution

Redes neuronales para predecir si existe un camino auto-evitante que pasa por un conjunto de puntos dados en una cuadrícula

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El objetivo de este trabajo es simular polímeros fijados en una base, con un método llamado caminos auto-evitantes (SAW por sus siglas en inglés) en una cuadrícula. Estos son caminos que no se cruzan. El propósito es estudiar las propiedades físicas de los polímeros. De momento hemos desarrollado un programa que genera SAWs y con sus coordenadas dibujar y hacer animaciones en una página web usando JavaScript. El algoritmo usado parte de los puntos base y va añadiendo puntos vecinos adicionales que preservan la propiedad de SAW.

Este proyecto inicial sirvió de base para plantear un problema nuevo. Se sabe que encontrar SAW incluso con una longitud moderada usando la técnica anterior es un problema computacionalmente intenso. Más aún, encontrar varios de ellos al mismo tiempo. Así que estamos comenzando un nuevo enfoque en el que, partiendo de un conjunto de puntos en el enrejillado usamos una red neuronal entrenada para detectar si esos puntos probablemente tengan SAWs que los atraviesen.

En esta presentación discutiremos consideraciones sobre la generación de conjuntos de entrenamiento y prueba y de diseño de la red que mejoran su rendimiento.

Palabras clave: caminos autoevitantes, redes neuronales, polímeros

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