A continuation method for the computation of fracture surfaces in strain space

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In this talk we discuss the numerical aspects of computing the onset of cavitation–type material instabilities in solids. To model this phenomenon we use nonlinear elasticity to allow for the large, potentially infinite, stresses and strains involved in such deformations. On two previous papers, the authors introduced the concept of *volume derivative* which can be used to characterize the critical boundary displacements for cavitation. If a body occupies the region $\Omega \in \mathbb{R}^n$, and $\mathbf{Ax}$ for $\mathbf{x} \in \Omega$ represents an affine boundary displacement for such a body, then the volume derivative $G(\mathbf{A})$ measures the amount of energy (positive or negative) required to open a hole of unit volume in the given material. For a large class of materials, the *onset of cavitation–type material instabilities* can be characterized as the zero level set of G.

The bulk of the computational work in computing the zero level set of G is on the evaluation of the volume derivative $G(\mathbf{A})$ for many boundary displacements $\mathbf{A}$. Each evaluation of G requires the solution of a large-scale constrained optimization problem which we accomplish by combining three different numerical techniques, namely, regularization, penalization and a gradient flow iteration. This procedure for evaluating G forms the basis for the *continuation method* that we describe for approximating the zero level set of G. We present some preliminary results for two dimensional problems ($n = 2$) that show the effectivity of the continuation method for computing the zero level set of G versus the more direct approach of using a contour finding routine.

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